Information Cascades, AQRE, and Rational Expectations

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May 2020

Information cascade

Generalities on information cascades

"An information cascade occurs when it is optimal for an individual, having observed the actions of those ahead of him, to follow the behavior of the preceding individual without regard to his own information."

Bikhchandani, Hirschleifer, and Welch, 1992

Information cascades capture the idea that a group of rational decision makers can generate collective behaviors that seem, *prima facie*, irrational (e.g. financial bubbles, spread of false information, fashion cycles and fades...).

Simple game with information Cascades

Two urns: A=(2 red balls and 1 blue ball) and B=(1 red ball and 2 blue balls), and seven players (P0,P1,...,P6).

First, P0 chooses A with probability $\frac{1}{2}$ and B with probability $\frac{1}{2}$.

Then, each of the six remaining Players receives the payoff of 1 if they choose the urn chosen by Player 0, 0 otherwise.

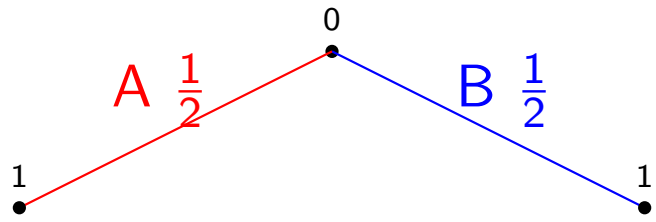
They play sequentially, i.e.

1. P1 draws a ball from the urn chosen by P0 and chooses an urn;
2. P2 observes P1's behavior, draws a ball from the urn chosen by P0, and chooses an urn.
3. P3 observes P1 and P2's behaviors, draws a ball from the urn chosen by P0, and chooses an urn; And so forth...

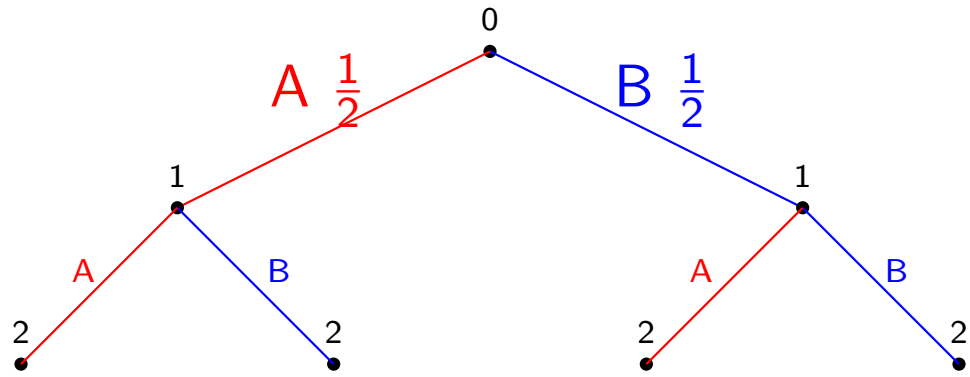
The Extensive Form Representation with three players



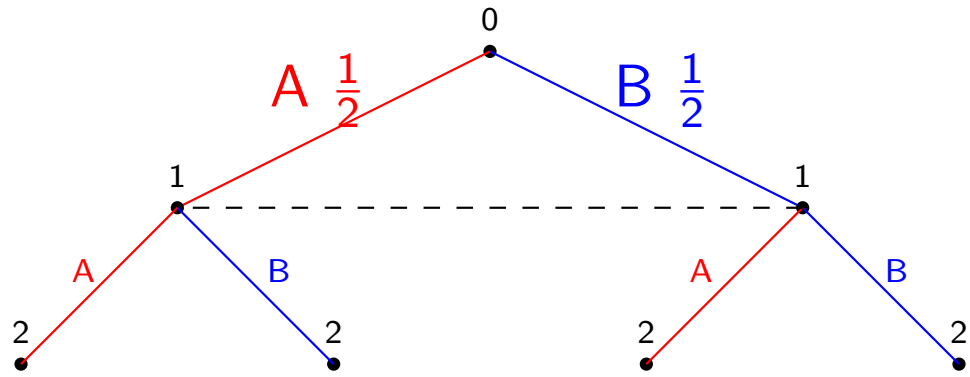
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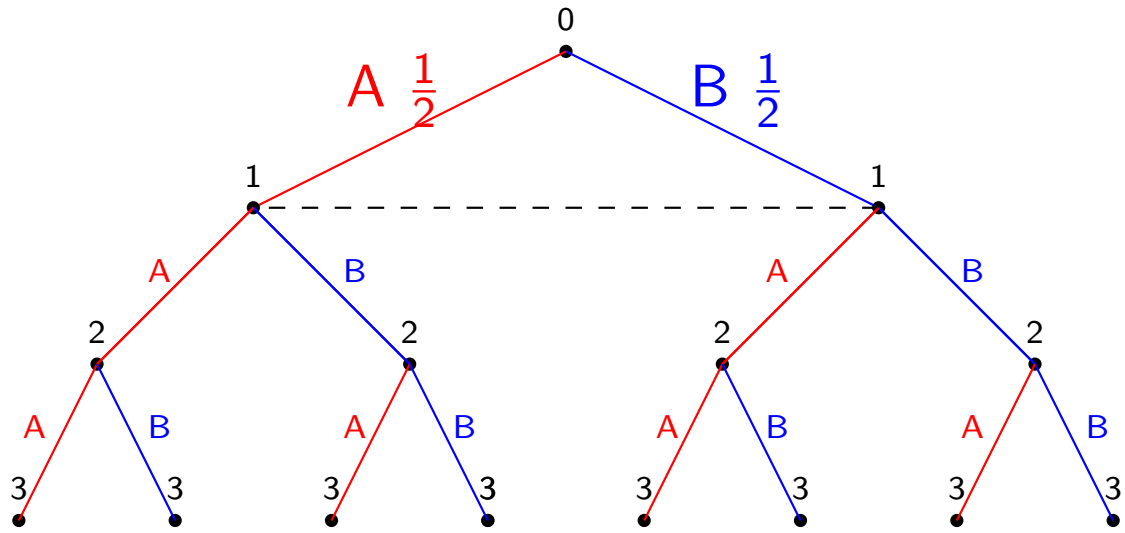
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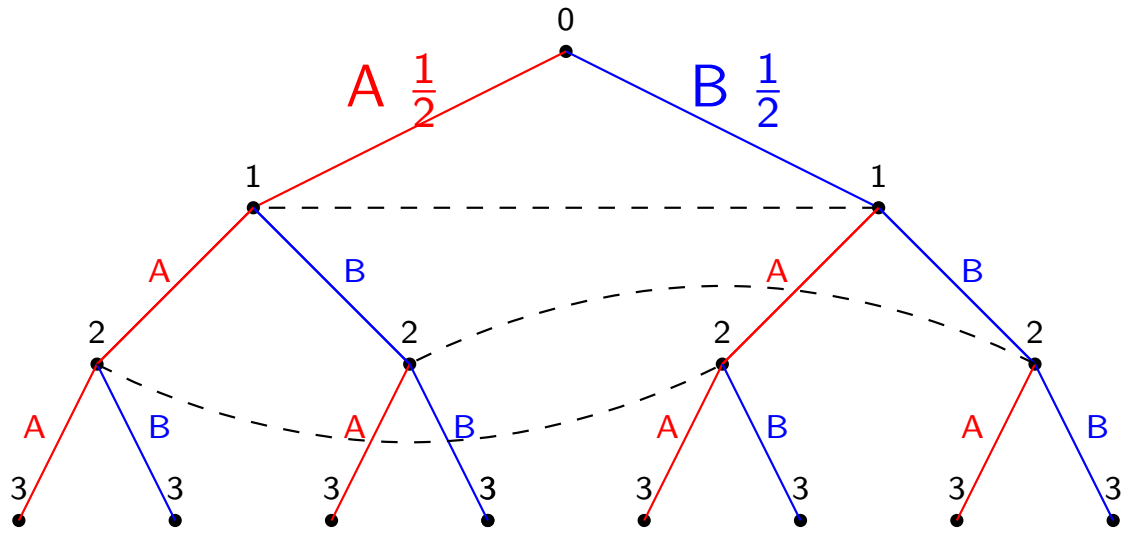
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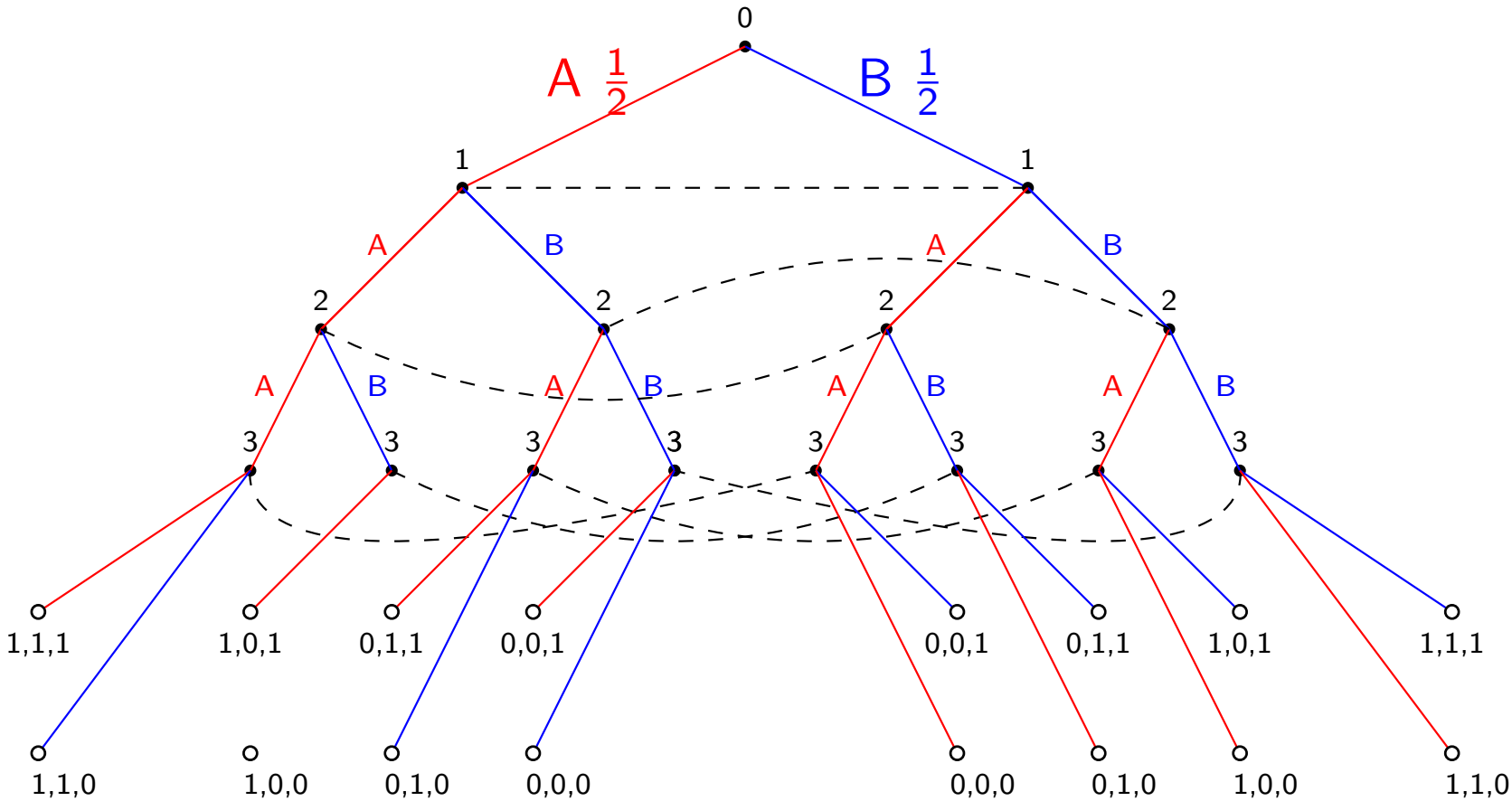
The Extensive Form Representation with three players



The Extensive Form Representation with three players



The Extensive Form Representation with three players



Intuition

If P1 and P2 choose A, then P3 chooses A, no matter what signal she receives.

1. P1 chooses A only if she draws a red ball, so since P2 and P3 believe that P1 is rational, they know that P1 has drawn a red ball.
2. If P2 receives also draws a red ball, then she chooses A for sure; if she draws a blue ball then she chooses A with probability $1/2$. So, given that P2 chooses A, for P3, the probability that she has drawn a red ball is $2/3$.
3. So, because P3 believes that P1 and P2 are rational, she believes that
   - the probability that both players have drawn a red ball is $2/3$;
   - the probability that P1 has drawn a red ball and P2 a blue ball is $1/3$.
4. Therefore, even if she were to receive the signal b, P3 would have, on the one hand, $(1+2/3)$ signals indicating the urn is A, on the other hand $(1+1/3)$ indicating that the right urn is B. Thus, she chooses A.

Notations

- A sequence of choices by several player is typically denoted $AABA....$
- h_i the previous choices observed by player i; E.g $h_3 = AB$.
- s_i (with $s = a$ or b) is the signal received by Player i: If Player i draws a red ball then $s_i = a_i$, otherwise $s_i = b_i$.
- q is the signal informativeness: the probability to draw a red ball (respectively a blue ball) in urn A (respectively in urn B).
- $\pi_i^{s_i}$ is the probability P0 has chosen A, given h_i and s_i, so

$$\pi_i^{s_i} = \mathbb{P}(A_0 | h_i, s_i)$$

Does predicts subjects behaviors in Lab?

Anderson and Holt (1997) tested this game experimentally with 6 players.

They conclude that

- "Individuals generally used information efficiently and followed the decisions of others when it was rational."
- "However, **some errors**, which tended to make subjects rely more **on their own private information.**"

In this course

Information cascades are relevant for us because they allow to deal with the following questions

- Can we explain the deviations from PBE with Quantal Response Equilibrium (AQRE)?
- Or should we explain the data with violation of Rational Expectations? I.e. the subjects do not weight symmetrically the source of information they have.
- Notably, do people assume their opponents' rationality; or do they systematically overweight their own information as opposed to the information they should infer from others behavior?

Paper 1:

Self-Correcting Information Cascades

Goeree, Palfrey, Rogers, and Mckelvey (2007)

Motivations

Pure Cascade vs craze:

- A pure cascade lasts until the end of the game.E.g BABAAAAAAAAA...AA
- A craze is an interrupted cascade. E.g. BAAAABBA

PBE predicts no craze while AQRE may be compatible with some crazes. Similarly, PBE predicts that subjects should have the same behavior as soon as the signal is informative.

The experiment

Same design as Anderson and Holt (1997), except that

- To observe some crazes they consider longer sequences with $T = 20$ or $T = 40$ subjects while Anderson and Holt there were only 6 subjects by sequences;
- They study more type of signals: q is either $5/9$ or $2/3$ while in Anderson and Holt it was fixed at $2/3$.

Violation of PBE

	q = 5/9 T = 20	q = 5/9 T = 40	q = 6/9 T = 20	q = 6/9 T = 40
# sequences	116	56	90	60
# sequences with pure cascades	5	0	12	8
# sequences without cascades	0	0	0	0
# sequences with broken cascades	111	56	78	52

Table 1. *Percentages of (broken) cascades in our data.*

What AQRE predicts?

Each individual t privately observes a payoff disturbance for each choice, denoted ϵ_i^A and ϵ_i^B, where the $\epsilon_i = \epsilon_i^A - \epsilon_i^B$ is assumed to obey a *Logistic distribution* with parameter λ.

Expected payoffs are:

$$\mathbb{E}(A|h_i, s_i) = \pi_i^{s_i}(h_i) + \epsilon_i^A \text{ and } \mathbb{E}(B|h_i, s_i) = 1 - \pi_i^{s_i}(h_i) + \epsilon_i^B$$

and, therefore **the probability of choosing** A **given** h_i **and** s_i is

$$\mathbb{P}(\mathbb{E}(A|h_i, s_i) - \mathbb{E}(B|h_i, s_i) > 0) = \mathbb{P}(\epsilon_i > 1 - 2.\pi_i^{s_i}(h_i)) = \frac{1}{1 + exp(\lambda.(1 - 2.\pi_i^{s_i}(h_i)))}$$

What AQRE predicts?

In quantal response equilibrium, **all actions are chosen with positive probability**; therefore, we expect

1. ... the probability that a cascade breaks in to be increasing with t: the longer the sequence the likelier it is to break in;
2. ... the number of cascades to be increasing in T.

What AQRE predicts?

In AQRE, **the closer from PBE the likelier is a strategy to be chosen,** so the DM responds to the informativeness of their signal. Thus, we expect

1. ... the probability that a pure cascade breaks to decrease in q.
2. ... incorrect cascades to be shorter than correct cascades and reverse more frequently.
3. ... later cascades to be likelier to be correct than earlier ones.

- The probability that a cascade breaks in is increasing with t: the longer the sequence the likelier it is to break in;
- The probability that a pure cascade breaks in decreases in q.

TABLE 4

Percentages of pure cascades by treatment

	Our data				AH data			QRE–BRF			
	$q = 5/9$		$q = 6/9$		$q = 6/9$	Nash		$q = 5/9$		$q = 6/9$	
	$T = 20$ $M = 116$	$T = 40$ $M = 56$	$T = 20$ $M = 90$	$T = 40$ $M = 60$	$T = 6$ $M = 45$	$q = 5/9$	$q = 6/9$	$T = 20$	$T = 40$	$T = 20$	$T = 40$
First 6 (%)	36	36	50	53	64	98	99	27	26	44	44
First 10 (%)	15	14	31	42		100	100	9	11	28	27
First 20 (%)	4	2	13	32		100	100	1	1	15	14
First 40 (%)		0		13		100	100		0		7

TABLE 5

Percentages of temporary cascades by treatment

	Our data				AH data			QRE–BRF			
	$q = 5/9$		$q = 6/9$		$q = 6/9$	Nash		$q = 5/9$		$q = 6/9$	
	$T = 20$ $M = 116$	$T = 40$ $M = 56$	$T = 20$ $M = 90$	$T = 40$ $M = 60$	$T = 6$ $M = 45$	$q = 5/9$	$q = 6/9$	$T = 20$	$T = 40$	$T = 20$	$T = 40$
First 6 (%)	58	55	42	38	27	0	0	64	62	50	48
First 10 (%)	84	84	69	55		0	0	88	87	71	71
First 20 (%)	96	98	87	68		0	0	99	99	85	86
First 40 (%)		100		87		0	0		100		93

What AQRE predicts?

- The number of cascades is increasing in T.

		q = 5/9		q = 6/9	
		N = 20 M = 116	N = 40 M = 56	N = 20 M = 90	N = 40 M = 60
average number cascades	Our Data	3.47	7.54	2.99	3.73
	QRE - BRF	3.85	7.31	2.81	4.19
	Nash	1.00	1.00	1.00	1.00
average length cascades	Our Data	2.43	2.00	3.27	7.83
	QRE - BRF	1.59	2.04	3.79	6.50
	Nash	17.32	37.25	17.43	37.44

Table 6. *Number and lengths of cascades by treatment.*

- Later cascades are more likely to be correct than earlier ones.
- Reverse cascades are usually self-correcting, and repeat cascades are usually correct.

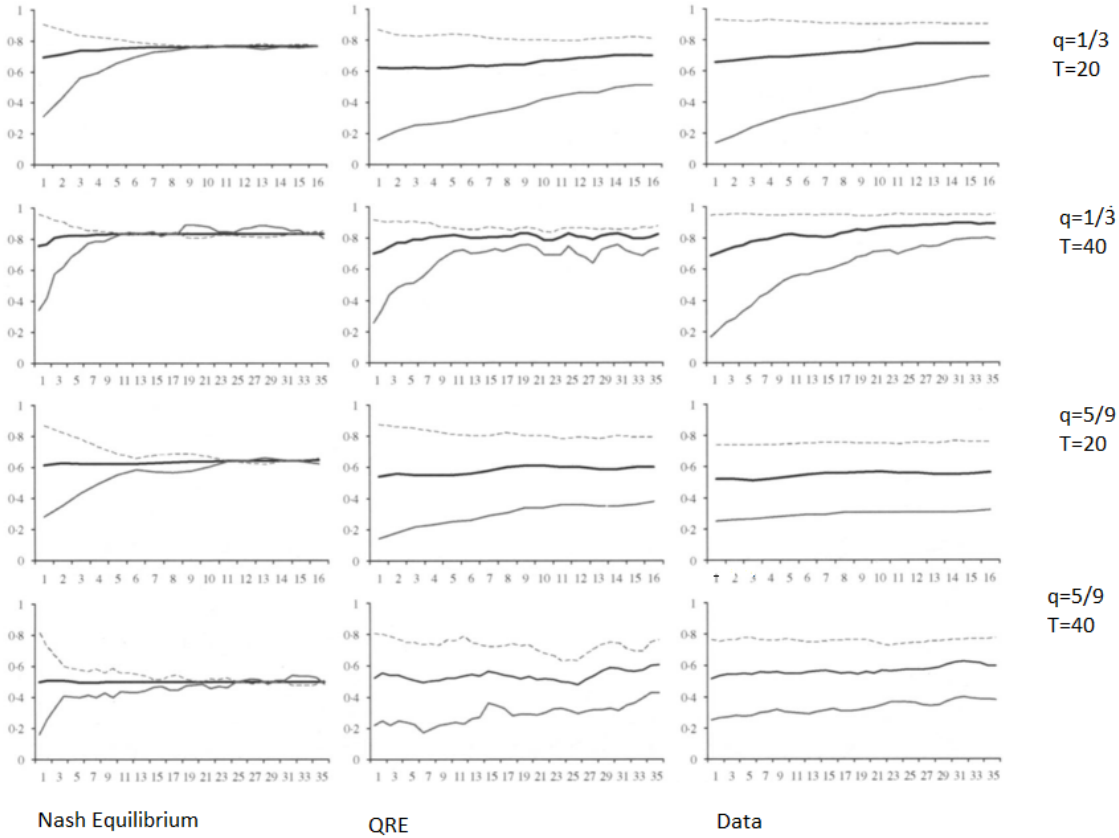
TABLE 9

Transitions between correct and incorrect cascades in our data

		$T = 20$			$T = 40$		
$q = 5/9$	From\To	Correct	Incorrect	From\To	Correct	Incorrect
	Correct	92·7%	7·3%	Correct	93·6%	6·4%
	Incorrect	22·7%	77·3%	Incorrect	11·0%	89·0%
# Correct	Total = 252	Start = 65	Final = 78	Total = 237	Start = 29	Final = 34
# Incorrect	Total = 151	Start = 51	Final = 38	Total = 185	Start = 27	Final = 22
$q = 6/9$	From\To	Correct	Incorrect	From\To	Correct	Incorrect
	Correct	91·4%	8·6%	Correct	98·7%	1·3%
	Incorrect	30·5%	69·5%	Incorrect	20·0%	80·0%
# Correct	Total = 197	Start = 66	Final = 71	Total = 186	Start = 48	Final = 52
# Incorrect	Total = 72	Start = 24	Final = 19	Total = 38	Start = 12	Final = 8

What AQRE predicts?

- Later cascades are more likely to be correct than earlier ones.
- More informative signal lead to faster learning. That is, the rate of increase in the *ex ante* probability of a correct decision as t increases is increasing in q.



What feature of the data is not captured AQRE?

They assume that the DM uses the same λ to make her decision and to learn from others meaningthe DMs treat symmetrically their signal: **Rational Expectations**.

- AQRE implies that indifferent agents should randomize uniformly their decision. Nevertheless, the data show that 85% of the indifferent subjects are likelier to follow their own signal.

As suggested by (Kahneman and Tversky, 1973), this assumption is often violated in lab. We call that *The Base Rate Fallacy*.

Paper 2:
Limited Depth of Reasoning and Failure of Cascade Formation in the Laboratory

Kübler and Weizsäcker (2004)

The experiment

The same as Anderson and Holt (1997), except that

- They introduce a separate stage for each player at which she is asked whether or not she wants to buy a signal.
- They also introduce treatments in which subjects were given the information who of the previous subjects had obtained a signal.

The motivations:

PBE predicts that P1 should buy the signal ; every other player must not buy the signal and chooses that same action as P1.

If P2, P3, ..., P6 were to buy the signal, this could either be explained in two ways,

- Either they think others are less rational, contradicting **Rational Expectations**
- or they think they did not buy the signal.

Adding the possibility for the DMs to know whether their opponents bought the signal allows for testing which of these two explanation applies.

The Model

- $u(A, s_i, h_i) = \pi_i^{s_i} - c$ if they bought the signal; $u(A, s_i, h_i) = \pi_i^{s_i}$ otherwise.
- The probability of choosing A is

$$\mathbb{P}(A|s_i, h_i, \lambda_1) = \frac{exp\big(\lambda_1 u(A, s_i, \alpha_i)\big)}{exp\big(\lambda_1 u(A, s_i, h_i) + exp\big(\lambda_1 u(B, s_i, h_i)\big)\big)}$$

- When deciding whether to buy a signal or not, subjects are assumed to anticipate their own decision probabilities when choosing an urn, and to calculate the expected utility of buying this signal $\hat{u}(\text{"}buy\text{"}, \alpha_i)$. So the probability of buying a signal is then

$$\mathbb{P}(\text{"}buy\text{"}; |s_i, \alpha_i, \lambda_1) = \frac{exp\big(\lambda_1 \hat{u}(A, s_i, h_i)\big)}{exp\big(\lambda_1 \hat{u}(\text{"}Buy\text{"}, h_i)\big) + exp\big(\lambda_1 \hat{u}(\text{"}NoBuy\text{"}, h_i)\big)}$$

The Model

- The subject is aware that all other subjects follow the logistic decision process described above,...
- she attributes a possibly different precision parameter to the decisions of her opponents: λ_2 instead of λ_1.

Thereby, the "rational expectations" assumption of the AQRE, that subjects are informed about the error rate of their opponents, is avoided and can be tested

- Analogously, when a subject considers the reasoning that others apply when thinking about third subjects, we allow for a third parameter λ_3;
- ... and so on until λ_6.

Results

1. The subjects learn from observing their predecessors' decisions, but they fail to realize that other subjects also learn from observing their respective predecessors.
2. The subjects bought even more signal when they can observe whether their opponents bought the signal or not. Thus, the results cannot be explained by distrust.
3. AQRE can fit the data as soon as it relaxes *Rational Expectations Assumption*.

TABLE 2

Response precisions estimated from the experimental data

Data	Pooled	HC	LC	HCHI	NC
λ_1	10·45 (0·000, 0·000)	11·36 (0·000, 0·139)	8·19 (0·000, 0·941)	12·97 (0·000, 0·000)	10·84 (0·000, 0·000)
λ_2	5·94 (0·000, 0·000)	8·12 (0·000, 0·000)	8·31 (0·000, 0·000)	4·71 (0·000, 0·000)	3·77 (0·000, 0·000)
λ_3	1·65 (0·795, 0·194)	1·29 (0·994, 0·641)	2·44 (0·905, 0·575)	0·00 (1·000, 0·970)	0·00 (1·000, 0·996)
λ_4	0·00 (1·000, 0·968)	0·00 (1·000, 0·975)	0·61 (1·000, 0·908)	—	—
λ_5	—	—	373·32 (1·000, 0·981)	—	—
λ_6	—	—	0.00 (0·996, —)	—	—
ll^*	−2045·9389	−518·1821	−470·2698	−523·7681	−464·9299

Note: Numbers in parentheses are (i) the marginal level of significance for the parameter to be different from zero, and (ii) the marginal level of significance for the parameter to be different from the parameter on the next-higher level.

Paper 3:
Do we Follow Others when we Should? A simple Test of Rational Expectations

Weizsäcker (2010)

Motivations

- Meta-analysis of Anderson and Holt (1992)'s like experiment.
- Given the deviation of PBE and AQRE, to what extend DMs choose the *empirically optimal* action: i.e., the action that is ex post optimal in the majority of cases under identical condition?
- Test of the rational expectations hypothesis?

Variables

- $contradict_i = \begin{cases} 1 & \text{if } i \text{ contradicts her own signal} \\ 0 & \text{otherwise} \end{cases}$
  E.g., chooses A with signal $s_i = b$.
- $\mathbf{I}_i = (h_i, s_i)$ = participants' information where s_i is the signal received by i and h_i the previous decision before i.
- $S(i)$ = set of individuals having the same information as i, i.e., the js' such that $\mathbf{I}_i = \mathbf{I}_j$,
- $mean_{pay}|contradict_i = \frac{1}{|S(i)|} \sum_{j \in S(i)} \mathbf{1}(s_j \text{"is wrong"})$
- $prop_contradicts_i = \frac{1}{|S(i)|} \sum_{j \in S(i)} \mathbf{1}\ (contradict_i = 1)$

Results: Reluctance to contradict one's signal

- The fitted line is increasing: participants at least roughly respond to the incentives that they face;
- Large discrepancy between observations where $mean_{pay}|contradict_i$ is small versus large;
- Had the subjects make the payoff maximizing choice in each case, the regression line would be an $S-$shaped line through (0.5,0.5). Here to have the participants contradicting their signal with more than 0.5 probability, the $mean_{pay}|contradict_i$ must be at 0.68.

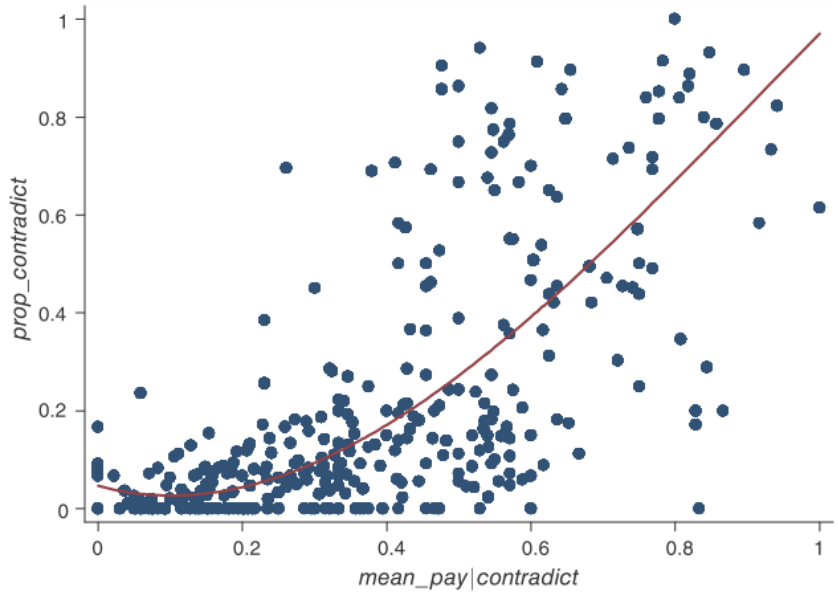


FIGURE 1. PROPORTION OF CONTRADICTING OWN SIGNAL ACROSS ALL OBSERVATIONS WITH $sitcount_i > 10$

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TABLE 2—FREQUENCY OF CONTRADICTING THE OWN SIGNAL

	Contradict	
	OLS (1)	IV (2)
mean_pay|contradict[1]	0.529***	0.945***
	(0.037)	(0.036)
constant	0.023	−0.135***
	(0.014)	(0.012)
H_0: (0.5, 0.5)	−28.58	−19.38
Observations	21,670	21,670
R^2	0.136	0.052

Notes: Data includes cases with $sitcount_i > 1$. Robust standard errors in parentheses, clustered by $group_i$.
***Significant at the 1 percent level.

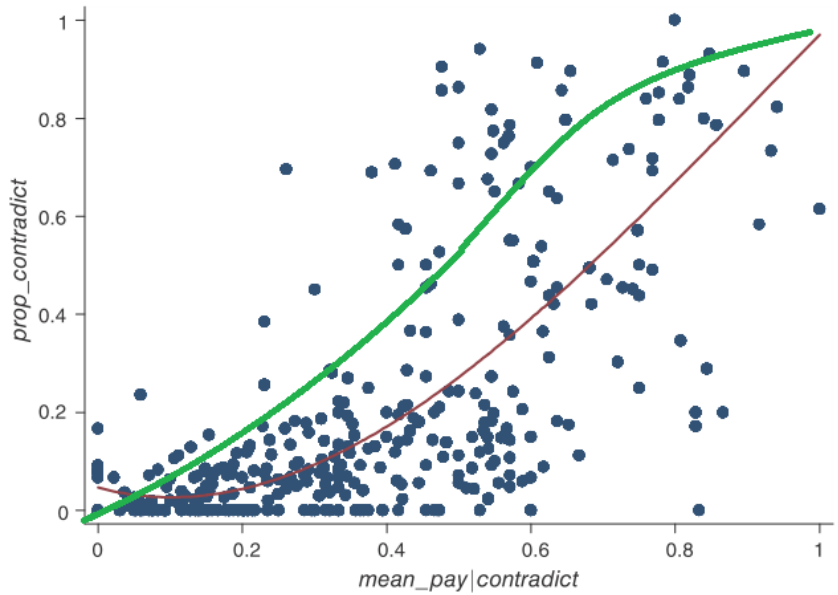


FIGURE 1. PROPORTION OF CONTRADICTING OWN SIGNAL ACROSS ALL OBSERVATIONS WITH $sitcount_i > 10$

Results: Testing rational expectations

A player exhibit best responses to rational expectations if action A is chosen if and only if

$$\mathbb{E}(\pi|A, I_i) - \mathbb{E}(\pi|B, I_i) + \epsilon_i \geq 0$$

$$\begin{aligned} p_i(A|I_i) &= p_i(\mathbb{E}(\pi|A, I_i) - \mathbb{E}(\pi|B, I_i) + \epsilon_i \geq |I_i) \\ &= 1 - F(\mathbb{E}(\pi|A, I_i) - \mathbb{E}(\pi|B, I_i)) \\ &= 1 - F(2\mathbb{E}(\pi|A, I_i) - 1) \end{aligned}$$

One can state that

$$\mathbb{E}(\pi|A, I_i) = mean_{pay}|A_i = \frac{1}{|S(i)|} \sum_{j \in S(i)} \mathbf{1}(A = A_0)$$

The tested model is then

$$p_i(A|I_i) = \alpha + \beta f(mean_{pay}|A_i) + \gamma d_i$$

If d_i describes information in I_i - e.g whether the DM decides late or not - then the model predicts that $\gamma = 0$. The reason is that if it were correct all relevant information in I_i should already been contained in $mean_{pay}$.

Exercice: Show that if P1 and P2 choose A then so is P3.

Assume P1 and P2 chooses A.

1. Show that $\pi_1^{a_1} = \frac{2}{3}$. Deduce from that P1 chooses A if an only if she draws a red ball.
2. Explain why $\pi_2^{a_2} = \mathbb{P}(A_0|a_1 a_2)$.
3. Show that $\pi_2^{a_2} = 1$ and $\pi_2^{b_2} = \frac{1}{2}$. Deduce from that if P2 draws a red ball then she chooses A for sure and that if she draws a red ball then she chooses A with probability $\frac{1}{2}$.
4. Show that if P1 and P2 choose A, then the probability that P2 has drawn a red ball is $\frac{2}{3}$, i.e. $\mathbb{P}(a_2|A_1 A_2) = \frac{2}{3}$.
5. Explain why

$$\pi_3^{a_3} = \mathbb{P}(a_2|A_1 A_2).\mathbb{P}(A_0|a_1 a_2 a_3) + \mathbb{P}(a_2|A_1 A_2).\mathbb{P}(A_0|a_1 b_2 a_3)$$

and

$$\pi_3^{b_3} = \mathbb{P}(a_2|A_1 A_2).\mathbb{P}(A_0|a_1 a_2 b_3) + \mathbb{P}(a_2|A_1 A_2).\mathbb{P}(A_0|a_1 b_2 b_3)$$

6. Compute $\pi_3^{a_3}$ and $\pi_3^{b_3}$. Deduce that, whatever the signal she receives, P3 chooses A when P1 and P2 choose A.

(The solutions of this exercice are given at the end the slides)

Exercice: solutions

1. If Player 1 draws a red ball, then, Applying Bayes rule, we have that

$$\pi_1^{a1} = \frac{\frac{2}{3}\cdot\frac{1}{2}}{\frac{2}{3}\cdot\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{2}} = \frac{2}{3}$$

Thus,

$$\mathbb{E}(A_0|a_1) = \pi_1^{a1} > (1 - \pi_1^{a1}) = \mathbb{E}(B_0|a_1)$$

so, P1 chooses A.

2. Since P2 **believes in P1's rationality**, P2 deduces from P1's choice that she has drawn a red ball. Thus, $\pi_2^{a2} = \mathbb{P}(A_0|A_1 a_2) = \mathbb{P}(a_1|A_1) = \mathbb{P}(A_0|a_1 a_2)$
3. Applying Bayes rule we have

$$\pi_2^{a2} = \mathbb{P}(A_0|a_1 a_2) = \frac{\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{1}{2}}{\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{2}}$$

and

$$\pi_2^{b2} = \mathbb{P}(A|a_1 b_2) = \frac{\frac{1}{3}\cdot\frac{2}{3}\cdot\frac{1}{2}}{\frac{1}{3}\cdot\frac{2}{3}\cdot\frac{1}{2} + \frac{2}{3}\frac{1}{3}\cdot\frac{1}{2}} = \frac{1}{2}$$

Thus

$$\mathbb{E}(A|Aa_2) = \pi_2^{a2} = \frac{4}{5} > \frac{1}{5} = \mathbb{E}(B|A, a)$$

and

$$\mathbb{E}(B_0|A_1 b_1) = \pi_2^{b2} = \frac{1}{2} = \mathbb{E}(B|Ab_2)$$

In case of P1 and P2 choosing A, P3, who **believes in P1 and P2's rationality**, deduces that P1 has drawn a red ball for sure and P2 has drawn a red ball with probability of $2/3$ or a blue ball with probability of $1/3$. Indeed,

$$\mathbb{P}(a_2|AA) = \frac{\mathbb{P}(AA|a_1, a_2)\mathbb{P}(a_1, a_2)}{\mathbb{P}(AA|a_1a_2)\mathbb{P}(a_1a_2) + \mathbb{P}(AA|a_1b_2)\mathbb{P}(a_1b_2)} = \frac{1 \times \frac{1}{4}}{1 \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4}} = \frac{2}{3}$$

Therefore,

$$\mathbb{E}(A|AAa_3) = \mathbb{P}(a_2|A).\mathbb{P}(A|Aa_3) + (1 - \mathbb{P}(a_2|Aa_2a_3)).\mathbb{P}(U_1|Ab_2a_3) =$$

$$\frac{2}{3}.\frac{(\frac{2}{3})^3.\frac{1}{2}}{(\frac{2}{3})^3.\frac{1}{2} + (\frac{1}{3})^3.\frac{1}{2}} + \frac{1}{3}.\frac{(\frac{1}{3})^3.\frac{1}{2}}{(\frac{2}{3})^3.\frac{1}{2} + (\frac{1}{3})^2.\frac{1}{2}} = \frac{17}{27} > \frac{17}{27} = \mathbb{E}(B|AAa_3)$$

and

$$\mathbb{E}(A|AAb_3) = \mathbb{P}(a_2|AA).\mathbb{P}(U_1|Aa_2b_3) + (1 - \mathbb{P}(a_2|aA)).\mathbb{P}(U_1|r_1, b_2, r_3) =$$

$$\frac{2}{3}.\frac{\frac{1}{3}.(\frac{2}{3})^2.\frac{1}{2}}{\frac{1}{3}.(\frac{2}{3})^2.\frac{1}{2} + (\frac{1}{3})^2.\frac{2}{3}.\frac{1}{2}} + \frac{1}{3}.\frac{\frac{2}{3}.(\frac{1}{3})^2.\frac{1}{2}}{\frac{2}{3}.(\frac{1}{3})^2.\frac{1}{2} + (\frac{2}{3})^2.\frac{1}{3}.\frac{1}{2}} = \frac{5}{9} > \frac{4}{9} = \mathbb{E}(B|AAb_3)$$

So, in any case Player 3 chooses A, even when she draws a blue ball.