BEHAVIORAL ECONOMICS · LECTURE 2

Revealed preference and the measurement of utility

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Where we are in the course

The assumptions of the Robinson model

0. **Consistent preferences:** Robinson's choices can be described by a utility function u
1. **Costless optimization:** he always finds the option that maximizes u
2. **Correct beliefs & expected utility:** he knows p and maximizes expected utility
3. **Absolute, self-regarding consumption:** only his own (b, y) enters u
4. **Stable tastes:** the same u in both periods

Today we make Assumption 0 precise: which **choices** are consistent with maximizing *some* utility function? The answer gives testable conditions. Lecture 3 extends them to random choices; Lecture 4 confronts them with the evidence.

Before we start: Robinson's lunches

The data: We observe choices from different menus:

Day	Available dishes	Chosen dish
1	Pizza, Burger	Pizza
2	Burger, Avocado	Burger
3	Avocado, Pizza	Pizza
4	Pizza, Burger, Avocado	Pizza

Your task

1. Can these choices come from maximizing a utility function u? Give one.
2. Is your u the only one? How many are there?
3. Invent a table of choices that **no** utility function can explain.

By the end of this lecture you should be able to

- Explain why utility cannot be measured directly, and what **revealed preference** does instead.
- State and prove: binary choices are explained by a utility function **iff** they are complete and transitive.
- State **WARP** and prove that it characterizes choice functions that maximize a utility.
- Test budget data with **GARP** (Afriat's theorem).
- Discuss whether transitivity and completeness are requirements of **rationality**.

Roadmap

Measuring utility

Binary choice data

Choice functions and WARP

Budget data: Afriat's theorem

1

Measuring utility

There is no instrument for utility: revealed preference infers it from choices, and only up to a strictly increasing transformation.

The measurement problem

An old idea

The idea of grounding economics on a measure of utility dates back to the 19th century:

- **Philosophy:** utilitarianism (Bentham, J. S. Mill).
- **Economics:** the marginalist revolution (Jevons, Menger, Walras).

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Problem: there is no instrument for utility.

Jevons, 1871

> *We have no means of defining and measuring quantities of feeling, like we can measure a mile, or a right angle, or any other physical quantity. . . We can hardly form the conception of a unit of pleasure or pain, so that the numerical expression of quantities of feeling seems to be out of the question. (Jevons, 1871)*

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Revealed preference: do not measure utility; infer it from what people **choose**.

Back to Robinson's lunches

Two utility functions

Both utility functions explain the four choices of the opening table:

$$u(\mathsf{P}) = 10,\ u(\mathsf{B}) = 5,\ u(\mathsf{A}) = 2 \qquad \text{and} \qquad v(\mathsf{P}) = 5,\ v(\mathsf{B}) = 3,\ v(\mathsf{A}) = 2.$$

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Check

- Check day 4: $\max\{u(\mathsf{P}), u(\mathsf{B}), u(\mathsf{A})\} = u(\mathsf{P})$, and Pizza was chosen. ✓
- $v = f \circ u$ with $f(10) = 5,\ f(5) = 3,\ f(2) = 2$: f is **strictly increasing**.

In general

If u explains the choices, so does $f \circ u$ for **every** strictly increasing f.

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If u explains the choices, so does $f \circ u$ for **every** strictly increasing f.

Choices reveal a **ranking**, not intensities: utility is **ordinal**. "Pizza is twice as good as Burger" has no meaning in this theory.

2

Binary choice data

With choices between pairs, a utility function exists if and only if the choices are complete and transitive.

A dataset that no utility function can explain

Three choices

Choice	Available dishes	Chosen dish
1	Pizza, Burger	Pizza
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No utility function

Suppose some u explains these choices. Then in each choice, the dish chosen must have the **higher** utility of the two available dishes:

- Choice 1: Pizza over Burger, so $u(\mathsf{P}) > u(\mathsf{B})$
- Choice 2: Burger over Avocado, so $u(\mathsf{B}) > u(\mathsf{A})$
- Choice 3: Avocado over Pizza, so $u(\mathsf{A}) > u(\mathsf{P})$

Chaining the three inequalities:

$$u(\mathsf{P}) \underset{1}{>} u(\mathsf{B}) \underset{2}{>} u(\mathsf{A}) \underset{3}{>} u(\mathsf{P}),$$

so $u(\mathsf{P}) > u(\mathsf{P})$: **impossible**.

Why a cycle cannot be represented by a utility function

The idea

The choices form a **cycle**: Pizza beats Burger, Burger beats Avocado, Avocado beats Pizza. Numbers cannot represent a cycle. Whatever values we give to $u(\mathrm{P})$, $u(\mathrm{B})$ and $u(\mathrm{A})$, at least one of the three choices is contradicted.

Why a cycle cannot be represented by a utility function

The idea

The choices form a **cycle**: Pizza beats Burger, Burger beats Avocado, Avocado beats Pizza. Numbers cannot represent a cycle. Whatever values we give to $u(\mathrm{P})$, $u(\mathrm{B})$ and $u(\mathrm{A})$, at least one of the three choices is contradicted.

Question

Which **single** choice can you change so that a utility function exists?

Goal of this section: characterize **all** datasets that a utility function can explain.

Setup: binary choice data

Setup

- X: a **finite** set of alternatives, e.g. $X = \{\text{Pizza}, \text{Burger}, \text{Avocado}\}$.
- For now we observe only choices between **pairs**, and exactly **one** option is chosen.

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Two consequences

- Since one option is chosen, $\succ^\star$ is **asymmetric**: $x \succ^\star y \Rightarrow$ not $y \succ^\star x$.
- A pair that is never offered gives **no** observation: neither $x \succ^\star y$ nor $y \succ^\star x$.

Representation

Definition (revealed preference)

For $x, y \in X$: $x \succ^\star y$ iff x is chosen from $\{x, y\}$. The dataset is $\langle X, \succ^\star \rangle$.

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Definition (representation)

A utility function $u : X \to \mathbb{R}$ *represents* $\langle X, \succ^\star \rangle$ if, for all $x, y \in X$:

$$x \succ^\star y \iff u(x) > u(y).$$

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Understanding this notion of representation is absolutely fundamental in this course!!!

Transitivity is necessary

Definition (transitivity)

$\succ^\star$ is *transitive* if, for all $x, y, z \in X$: $x \succ^\star y$ and $y \succ^\star z \Rightarrow x \succ^\star z$.

Two questions

1. If some u represents $\succ^\star$, is $\succ^\star$ transitive?
2. If $\succ^\star$ is transitive, does some u represent it?

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Answer to 1: yes.

Proof.

Let u represent $\succ^\star$ and let $x \succ^\star y$, $y \succ^\star z$. Then $u(x) > u(y) > u(z)$, so $u(x) > u(z)$, hence $x \succ^\star z$. □

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The cycle of the previous slide is exactly a failure of transitivity.

Is transitivity enough? Not quite

Example

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- $\succ^\star$ is transitive (there is no chain to check).
- Suppose u represents it. Then $u(x) > u(y)$.
- No observation between x and z: neither $u(x) > u(z)$ nor $u(z) > u(x)$, so $u(x) = u(z)$.
- Likewise $u(y) = u(z)$. Hence $u(x) = u(y)$: contradiction.

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A representation reads every *missing* observation as a *tie*. Missing data can therefore make a transitive dataset impossible to represent. Fix: observe all pairs.

Completeness and the representation theorem

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Theorem (Binary choice)

*Let X be finite and $\succ^\star$ asymmetric. Then $\succ^\star$ is complete and transitive **if and only if** there is a utility function u that represents it, with $u(x) \neq u(y)$ for $x \neq y$.*

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Proof strategy

- "If": transitivity shown on the previous slides; completeness because $u(x) \neq u(y)$ means $x \succ^\star y$ or $y \succ^\star x$.
- "Only if": we need to **construct** u from the data (next slide).

Constructing the utility function

Idea

Count how many alternatives each option beats:

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Proof that u represents $\succ^\star$.

($\Rightarrow$) Let $x \succ^\star y$. By transitivity, x beats everything that y beats. And x beats y, which y does not beat (itself). So $u(x) > u(y)$.
($\Leftarrow$) If not $x \succ^\star y$, then by completeness $x = y$ or $y \succ^\star x$. By ($\Rightarrow$), $u(x) \leq u(y)$. $\square$

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The proof is constructive: the utility function is built **from choices alone**.

Allowing for indifference

Weak revealed preference

So far exactly one option is chosen. Now allow "both are fine": write $x \succsim^\star y$ if x is **weakly** chosen over y, and

$$x \sim^\star y \iff x \succsim^\star y \text{ and } y \succsim^\star x, \qquad x \succ^\star y \iff x \succsim^\star y \text{ and not } y \succsim^\star x.$$

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Two properties

- **Complete:** for all $x, y \in X$, $x \succsim^\star y$ or $y \succsim^\star x$.
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Theorem (Representation of a weak order)

*Let X be finite. A binary relation $\succsim$ on X is complete and transitive **iff** there is $u : X \to \mathbb{R}$ with $x \succsim y \iff u(x) \geq u(y)$ for all $x, y \in X$.*

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Proof: the same count, $u(x) = \#\{z \in X : x \succsim z\}$, works (Exercise 3).

Reflection: is transitivity a requirement of rationality?

Money pump

Suppose $x \succ y \succ z \succ x$. You own x and 10 euros.

- You prefer z to x: I sell you z in exchange for x and 1 euro.



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- You own x again, and 7 euros. Repeat until you are broke.



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Intransitive preferences can be **exploited**. Unlike completeness, transitivity is also **falsifiable**: a cycle in the data refutes it.

But is intransitivity always irrational?

Small differences

Coffee with n grains of sugar: you cannot tell n from $n+1$, so $c_0 \sim c_1 \sim \cdots \sim c_{1000}$; yet $c_0 \succ c_{1000}$. Indifference is not transitive (Luce, 1956).

Several criteria

Flats compared on rent, size and location, choosing the one better on 2 of 3 criteria:

	Rent	Size	Location
Flat 1	best	middle	worst
Flat 2	worst	best	middle
Flat 3	middle	worst	best

Then flat 2 $\succ$ 1 (size, location), 3 $\succ$ 2 (rent, location), 1 $\succ$ 3 (rent, size): a cycle.

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Limited **attention** and **cognitive costs** (Part 2) can produce similar patterns.

Reflection: completeness

Aumann (1962)

Of all the axioms of utility theory, the completeness axiom is perhaps the most questionable. Like others of the axioms, it is inaccurate as a description of real life; but unlike them, we find it hard to accept even from the normative viewpoint. Certain decisions might involve highly hypothetical situations, which he will never face in real life. . . or might be extremely complex, too complex for intuitive "insight," and our individual might prefer to make no decision at all. (Aumann, 1962)

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- Problem with forced choice: Indifference $\neq$ completeness?

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- **Taboo:** it is reasonable to refuse the choice of "who must die" among several people.
- **Incomparable:** Ranking two movies is often impossible.

Recap: binary choice

Data	Condition	Representation
Strict binary choices on all pairs	complete + transitive	$x \succ^\star y \iff u(x) > u(y)$
Weak binary choices on all pairs	complete + transitive	$x \succsim^\star y \iff u(x) \geq u(y)$

- Utility is **ordinal**: any strictly increasing transformation of u represents the same choices.
- Transitivity: falsifiable, defended by the money pump, yet not always irrational to violate.
- Completeness: demanding on data, hard to test, questionable even normatively.

"Utility maximization" with binary data = complete and transitive choices. Nothing more, nothing less.

3

Choice functions and WARP

With menus of more than two options, binary consistency is not enough: utility maximization is equivalent to one condition linking choices across menus, WARP.

What about these choices?

Four days at the canteen

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Diagnosis

- The binary choices (days 1–3) are complete and transitive: P $\succ^\star$ B $\succ^\star$ A.
- Yet no utility function explains all four days: $u(\mathsf{P}) \underset{1}{>} u(\mathsf{B}) \underset{4}{>} u(\mathsf{P})$.

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With menus of more than two options, binary consistency is **not enough**: we need a condition that links choices **across menus**.

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Definition (choice function)

A *choice function* is a map $C : \mathcal{A} \to X$ with $C(A) \in A$ for every $A \in \mathcal{A}$.

- $C(A)$ = the option chosen from menu A. The dataset is $\langle X, C \rangle$.
- **Richness:** for now we observe $C(A)$ for **every** menu A.

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Definition (rationalization)

A utility function u *rationalizes* C if, for every $A \in \mathcal{A}$, $C(A)$ is the unique maximizer of u on A:

$$C(A) = x \iff u(x) > u(y) \text{ for all } y \in A \setminus \{x\}.$$

The Weak Axiom of Revealed Preference

In the table

Pizza is chosen when Burger is available (day 1), **and** Burger is chosen when Pizza is available (day 4).

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In words: once x has been chosen while y was available, y is never chosen while x is available.

Check on the table

Take the menus of day 1 and day 4, $x = \text{P}$, $y = \text{B}$. Then $C(\{\text{P}, \text{B}\}) = \text{P}$ but $C(\{\text{P}, \text{B}, \text{A}\}) = \text{B}$: WARP is violated.

Representation of a choice function

Theorem

Let X be finite and C a choice function on $\mathcal{A}$. The following are equivalent:

(i) *C satisfies WARP.*

(ii) *There is a complete, transitive and asymmetric relation $\succ$ on X such that, for all $A \in \mathcal{A}$,*

$$C(A) = x \iff x \succ y \text{ for all } y \in A \setminus \{x\}.$$

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WARP is **all** that utility maximization implies for choice functions: a single, testable condition.

Proof: (ii) $\Rightarrow$ (i) and the revealed relation

(ii) $\Rightarrow$ (i)

Let $x \neq y$ in $A \cap B$ with $C(A) = x$, so $x \succ y$. If $C(B) = y$, then $y \succ x$, contradicting asymmetry. So $C(B) \neq y$.

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(i) $\Rightarrow$ (ii)

Define $\succ^\star$ from binary menus: $x \succ^\star y \iff x \neq y$ and $C(\{x, y\}) = x$.

- **Complete and asymmetric:** exactly one element is chosen from each pair.

Proof: (ii) $\Rightarrow$ (i) and the revealed relation

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- **Complete and asymmetric:** exactly one element is chosen from each pair.
- **Transitive:** let $x \succ^\star y$ and $y \succ^\star z$, and suppose not $x \succ^\star z$, i.e. $C(\{x, z\}) = z$. Then

$$C(\{x, y\}) = x, \qquad C(\{y, z\}) = y, \qquad C(\{x, z\}) = z,$$

and every possible $C(\{x, y, z\})$ violates WARP: x contradicts $C(\{x, z\}) = z$, y contradicts $C(\{x, y\}) = x$, z contradicts $C(\{y, z\}) = y$.

Proof (continued): C picks the $\succ^\star$-best element

C chooses the maximum of $\succ^\star$

It remains to show that $C(A) = x \iff x \succ^\star y$ for all $y \in A \setminus \{x\}$.

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- ($\Rightarrow$) Let $C(A) = x$ and $y \in A \setminus \{x\}$. Apply WARP to A and $B = \{x, y\}$: $C(\{x, y\}) \neq y$, so $C(\{x, y\}) = x$, i.e. $x \succ^\star y$.
- ($\Leftarrow$) Let $x \succ^\star y$ for all $y \in A \setminus \{x\}$, and let $z = C(A)$. If $z \neq x$, then by ($\Rightarrow$), $z \succ^\star x$, contradicting $x \succ^\star z$ and asymmetry. So $C(A) = x$.

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(ii) $\Leftrightarrow$ (iii)

By the binary-choice theorem, a complete, transitive, asymmetric $\succ$ is represented by a one-to-one u: $x \succ y \iff u(x) > u(y)$. Substitute in (ii). Conversely, given u, define $x \succ y \iff u(x) > u(y)$. $\square$

Allowing for indifference: choice correspondences

Definition (choice correspondence)

A **choice correspondence** maps each menu $A \in \mathcal{A}$ to a nonempty set of chosen options $C(A) \subseteq A$ (all the options the decision maker is happy with).

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WARP for choice correspondences

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Theorem (Arrow, 1959)

Let X be finite. C satisfies WARP ***iff*** *there is a complete and transitive $\succsim$ (equivalently, a utility u) with*

$$\begin{aligned} C(A) &= \{x \in A : \ x \succsim y \ \ \forall y \in A\} \\ &= \{x \in A : \ u(x) \geq u(y) \ \ \forall y \in A\}. \end{aligned}$$

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Example

$C(\{\mathsf{P}, \mathsf{B}\}) = \{\mathsf{P}, \mathsf{B}\}$ and $C(\{\mathsf{P}, \mathsf{B}, \mathsf{A}\}) = \{\mathsf{P}\}$ violates WARP ($x = \mathsf{B}$, $y = \mathsf{P}$).

A strong assumption: richness

How many menus?

We assumed that we observe choices from **all** menus. How many are there?

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- Even in the lab, observing all menus is infeasible beyond a handful of options.
- In the field, consumers face **specific** menus: those defined by prices and income.

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Real data are partial. Next: what can we test with the menus that markets actually offer?

Recap: choice functions

The result

- Menu data are summarized by a choice function C (or a correspondence, with indifference).
- **WARP:** if x is chosen when y is available, then y is never chosen when x is available.
- **Theorem:** WARP $\iff$ C maximizes a complete, transitive preference $\iff$ C maximizes a utility u.

Two lessons

- Binary consistency is not enough: WARP links choices **across** menus (the decoy effect breaks this link).
- The theorem needs **richness**: choices from all menus.

4

Budget data: Afriat's theorem

Markets reveal prices and bought bundles, not all menus: with finitely many budget choices, utility maximization is equivalent to GARP.

From menus to markets

What markets show

- In markets we do not see choices from arbitrary menus.
- We see **prices** and the **bundle bought**: each budget is a menu with infinitely many options.
- We see only a **finite** number of such choices.

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Robinson at the market

On T days we record:

- prices $\boldsymbol{p}_t = (p_t^b, p_t^y) \in \mathbb{R}^2_{++}$ of bourbon and yoga classes;
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Is the dataset $\{(\boldsymbol{p}_t, \boldsymbol{x}_t)\}_{t=1}^T$ consistent with maximizing a utility $u(b, y)$?

Setup: budget data

Budget

On day t (income = what he spends):

$$B_t = \{\boldsymbol{x} \in \mathbb{R}^2_+ : \ \boldsymbol{p}_t \cdot \boldsymbol{x} \leq \boldsymbol{p}_t \cdot \boldsymbol{x}_t\}, \qquad \boldsymbol{p}_t \cdot \boldsymbol{x} = p_t^b \, b + p_t^y \, y.$$

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Definition (rationalization)

A utility function $u : \mathbb{R}^2_+ \to \mathbb{R}$ *rationalizes* the data if, for every t, $u(\boldsymbol{x}_t) \ge u(\boldsymbol{x})$ for all $\boldsymbol{x} \in B_t$.

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A necessary restriction

- Without further restrictions every dataset is rationalized: take u **constant**!
- We require u to be **locally non-satiated**: near every bundle there is a strictly better one (true if u is strictly increasing).

Revealed preference from budgets

Definitions

- **Directly revealed preferred:** $\boldsymbol{x}_t \succsim^D \boldsymbol{x}_s$ iff $\boldsymbol{p}_t \cdot \boldsymbol{x}_t \geq \boldsymbol{p}_t \cdot \boldsymbol{x}_s$.
- **Strictly directly revealed preferred:** $\boldsymbol{x}_t \succ^D \boldsymbol{x}_s$ iff $\boldsymbol{p}_t \cdot \boldsymbol{x}_t > \boldsymbol{p}_t \cdot \boldsymbol{x}_s$.
- **Revealed preferred:** $\boldsymbol{x}_t \succsim^R \boldsymbol{x}_s$ iff there is a chain $\boldsymbol{x}_t \succsim^D \boldsymbol{x}_{t_1} \succsim^D \cdots \succsim^D \boldsymbol{x}_{t_k} \succsim^D \boldsymbol{x}_s$ (transitive closure).

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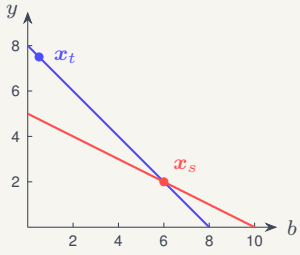
Reading $\succsim^D$

$\boldsymbol{x}_t \succsim^D \boldsymbol{x}_s$: $\boldsymbol{x}_s$ was affordable on day t, yet $\boldsymbol{x}_t$ was bought.

Reading $\succ^D$

$\boldsymbol{x}_t \succ^D \boldsymbol{x}_s$: $\boldsymbol{x}_s$ was affordable **with money to spare**. With local non-satiation, something strictly better than $\boldsymbol{x}_s$ was affordable, so $u(\boldsymbol{x}_t) > u(\boldsymbol{x}_s)$.

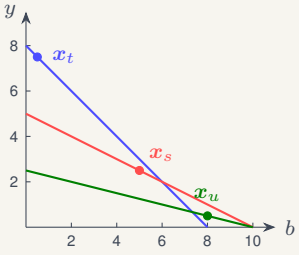
Illustration



$x_t \succsim^D x_s$



$x_t \succ^D x_s$



$x_t \succsim^R x_u$, not $\succsim^D$

Reading the figure

Axes: b = bourbon, y = yoga. Blue: B_t, prices $(1, 1)$, income 8. Red: B_s, prices $(1, 2)$, income 10. Green: B_u, prices $(1, 4)$, income 10.

- Left: $\boldsymbol{p}_t \cdot \boldsymbol{x}_s = 6 + 2 = 8$: affordable, no money to spare. Middle: $\boldsymbol{p}_t \cdot \boldsymbol{x}_s = 7.5 < 8$.
- Right: $\boldsymbol{x}_u = (8, 0.5)$ costs $8.5 > 8$ on day t, but $9 < 10$ on day s: $\boldsymbol{x}_t \succ^D \boldsymbol{x}_s \succ^D \boldsymbol{x}_u$.

GARP and Afriat's theorem

Generalized Axiom of Revealed Preference (GARP)

For all s, t:
$\boldsymbol{x}_t \succsim^R \boldsymbol{x}_s \implies$ not $\boldsymbol{x}_s \succ^D \boldsymbol{x}_t$.

In words: no chain of revealed preferences that ends with a **strict** direct reversal.

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With finitely many observations, continuity and concavity add **no** testable content: **GARP is everything**.

Worked example: testing GARP

Day	Prices $\boldsymbol{p}_t$	Bundle $\boldsymbol{x}_t$	Spending	Other bundle costs
1	$(1, 2)$	$(1, 4)$	$1 + 8 = 9$	$\boldsymbol{p}_1 \cdot \boldsymbol{x}_2 = 4 + 2 = 6$
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Direct revealed preferences

- Day 1: $\boldsymbol{x}_2$ cost $6 < 9$: $\boldsymbol{x}_1 \succ^D \boldsymbol{x}_2$.
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Verdict

So $\boldsymbol{x}_1 \succsim^R \boldsymbol{x}_2$ **and** $\boldsymbol{x}_2 \succ^D \boldsymbol{x}_1$: GARP fails. Robinson buys more of whatever is **expensive**, and could have had the other bundle with money to spare both times.

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So $\boldsymbol{x}_1 \gtrsim^R \boldsymbol{x}_2$ **and** $\boldsymbol{x}_2 \succ^D \boldsymbol{x}_1$: GARP fails. Robinson buys more of whatever is **expensive**, and could have had the other bundle with money to spare both times.

If instead he had bought $\boldsymbol{x}_1 = (4,1)$ on day 1 and $\boldsymbol{x}_2 = (1,4)$ on day 2, neither bundle is affordable on the other day (both cost 9 > 6): no revealed preference at all, GARP holds trivially.

Is GARP satisfied in practice?

Budget experiments

- Subjects choose many bundles on randomly drawn budget lines (often on a tablet).
- Choi et al. (2014) ran this with a large sample of the Dutch population:
  - most subjects are **close** to consistent, but rarely perfectly;
  - consistency is higher among the young, the more educated and the richer, and correlates with household wealth.

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How close? Afriat's efficiency index

The largest $e \leq 1$ such that the data pass GARP when every budget is shrunk to $e \cdot \boldsymbol{p}_t \cdot \boldsymbol{x}_t$. $e = 1$: consistent; $e = 0.9$: "10% of income wasted".

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Consistency is a matter of **degree**, and it can be **measured**, which turns Assumption 0 into an empirical variable.

Recap: what consistency requires

Data	Testable condition	Utility representation
Binary choices on all pairs	Complete + transitive	$x \succ^\star y \iff u(x) > u(y)$
Choices from all menus	WARP	$C(A) = \arg\max_{x \in A} u(x)$
Budget choices (finite)	GARP	u continuous, increasing, concave

Each kind of data comes with **one testable axiom** equivalent to utility maximization. Lecture 3: the same question when choices are random. Lecture 4: experiments that violate them.

Exercises

Exercise 1 · Easy

On $X = \{x, y, z\}$: $C(\{x, y\}) = x$, $C(\{y, z\}) = y$, $C(\{x, z\}) = x$. Which values of $C(\{x, y, z\})$ satisfy WARP? Give a utility function that rationalizes C.

Exercise 2 · Intermediate

Robinson's records: day 1, $\boldsymbol{p}_1 = (1, 1)$, $\boldsymbol{x}_1 = (3, 1)$; day 2, $\boldsymbol{p}_2 = (1, 3)$, $\boldsymbol{x}_2 = (1, 2)$. Find all direct revealed preferences; does GARP hold? Same questions if on day 2 he had bought $(4, 1)$.

Exercise 3 · Intermediate

Let X be finite and $\succsim$ complete and transitive. Show that $u(x) = \#\{z \in X : x \succsim z\}$ represents $\succsim$.

Exercise 4 · Advanced

Let X be finite and $\succ^\star$ asymmetric. Show that some u satisfies $x \succ^\star y \iff u(x) > u(y)$ **iff** $\succ^\star$ is *negatively transitive*: not $x \succ^\star y$ and not $y \succ^\star z \Rightarrow$ not $x \succ^\star z$. Use it to explain the "Is transitivity enough?" slide.

Discussion questions

The meaning of the axioms

- What are the main ideas behind transitivity and completeness?

Their empirical content

- Are these properties easy to test? What data do you need? Are they falsifiable?

Rationality

- Is it always irrational to violate them? How would you define rationality?
- Are they a **descriptive** or a **normative** benchmark?

Final take-aways

1. Utility cannot be measured directly; revealed preference infers it from **choices**, and only up to a strictly increasing transformation (**ordinal** utility).
2. For each type of data, utility maximization is equivalent to **one testable axiom**: completeness and transitivity (pairs), WARP (menus), GARP (budgets).
3. These axioms are falsifiable, so Assumption 0 is an **empirical** claim.
4. Whether violating them is **irrational** is a separate, normative question.

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