Review: Strategic Form Games and Nash Equilibrium

Summer 26

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Last lecture

Last lecture introduced the logic of the course:

$$\boxed{\text{Standard GT} \rightarrow \text{evidence} \rightarrow \text{behavioral model} \rightarrow \text{interpretation}}$$

We also distinguished four broad classes of games:

- simultaneous-move games,
- sequential games,
- games of incomplete information,
- repeated games.

Today

We begin with the benchmark for *simultaneous-move strategic interaction*:

- how to represent a game in strategic form,
- what rationality alone implies,
- what Nash equilibrium adds on top.

In the absence of a clear benchmark, subsequent deviations can be readily observed but are difficult to interpret.

Today

We begin with the benchmark for *simultaneous-move strategic interaction*:

- how to represent a game in strategic form,
- what rationality alone implies,
- what Nash equilibrium adds on top.

In the absence of a clear benchmark, subsequent deviations can be readily observed but are difficult to interpret.

The lecture is meant to revise material, but it is still important and should not be ignored.

Learning goals

By the end of this lecture, you should be able to

- define a game in **strategic form** and identify its primitives;
- identify **strictly** and **weakly dominated** strategies;
- explain why iterated elimination requires more than own rationality;
- define **Nash equilibrium** formally and interpret it as mutual best response;
- find pure-strategy Nash equilibria in payoff matrices;
- solve a simple **Cournot** game by best-response reasoning;
- explain why this benchmark is indispensable, but not yet a full descriptive theory of observed play.

Strategic form

Review: Strategic Form Games and Nash Equilibrium

Why start with strategic form?

We first need a precise description of the strategic situation:

- Who are the players?
- What can each player do?
- What consequences follow from each combination of choices?

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These elements define a strategic form i.e.

- the lists the players: N
- the strategies available to each player: $(S_i)_{i \in N}$
- the payoff each player receives for every strategy profile: $(u_i)_{i \in N}$

This does *not* yet solve the game. It gives us the objects on which dominance, best responses, and Nash equilibrium will later operate.

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- S_i is player i's strategy set;
- $S = \prod_{i \in N} S_i$ is the set of strategy profiles;
- $u_i : S \to \mathbb{R}$ is player i's payoff function.

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For every profile s, the game specifies payoffs

$$u_i(s) = u_i(s_1, \ldots, s_n), \qquad i \in N.$$

Example: Cournot oligopoly in strategic form

Consider an oligopoly with firms $i = 1, \ldots, n$.

- Each firm chooses an output level $q_i \geq 0$.
- so the strategy set of firm i is $S_i = [0, \infty)$.

Given the total output be $Q = \sum_{j=1}^{n} q_j$,

- the inverse demand is $P(Q)$,
- firm i's cost function be $C_i(q_i)$.

Firm i's profit is then $\pi_i(q_1, \ldots, q_n) = q_i\, P(Q) - C_i(q_i)$.

So the Cournot game in strategic form is given by

$$\Gamma = (N, (S_i)_{i \in N}, (\pi_i)_{i \in N}) .$$

From economic story to strategic form

Two suspects are interrogated separately. Each has two feasible actions:

$$C = \text{cooperate by staying silent}, \qquad D = \text{defect by testifying}.$$

The strategic tension is:

- each player prefers the other to cooperate while she defects;
- mutual cooperation is better for both than mutual defection;
- but each player has an individual incentive to defect.

So the Prisoner's Dilemma combines

- conflict of interest at the individual level,
- with mutual gains from coordinated restraint.

This makes it a canonical benchmark for the gap between

- individually rational behavior and
- collectively desirable outcomes.

Prisoner's Dilemma in strategic form

Let each player have strategy set

$$S_1 = S_2 = \{C, D\}.$$

The payoff matrix is

	C	D
C	$(3, 3)$	$(0, 4)$
D	$(4, 0)$	$(1, 1)$

This matrix contains the three ingredients of strategic form:

- the players: player 1 and player 2;
- the strategy sets: $\{C, D\}$ for each player;
- the payoff functions: the ordered payoff pair in each cell.

Next question: does rationality rule out some strategies?

Dominance

Review: Strategic Form Games and Nash Equilibrium

From representation to reasoning

Strategic form tells us *what the game is*. The next question is:

What can rational players already rule out before we talk about equilibrium?

Consider the following game:

	L	M	R
T	$(4,2)$	$(3,1)$	$(2,0)$
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- against L, T yields 4 $\succ_1$ B yields 2;
- against M, T yields 3 and B yields 1;

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For player 1:

- against L, T yields 4 $\succ_1$ B yields 2;
- against M, T yields 3 and B yields 1;
- against R, T yields 2 and B yields 0.

So whatever player 2 does, he is strictly better with T.

(Since player 2 knows that player 1 is rational, he chooses L.)

Strict dominance

Definition (Strict dominance)

We say that, for player i the strategy s_i is strictly dominated by the stratgey s_i' if

$$u_i(s_i', s_{-i}) > u_i(s_i, s_{-i}) \qquad \text{for every } s_{-i} \in S_{-i}.$$

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A strictly dominated strategy yields a strictly lower payoff

- against every possible strategy profile of the opponents,
- regardless of what the player believes others will do.

$\Longrightarrow$ a rational player should never choose a strictly dominated strategy.

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This reasoning does *not* require belief in opponents' rationality nor correct beliefs.

It follows from rationality alone.

$\Longrightarrow$ payoff maximization

Why strict dominance matters

Strict dominance is the first and cleanest implication of rationality in a game.

If a strategy is strictly dominated, then

- it is never a best response;
- it can be ruled out without knowing what opponents will do;
- eliminating it simplifies the strategic problem.

This gives us a natural iterative procedure:

identify dominated strategies $\rightarrow$ remove them $\rightarrow$ check whether new dominance relations appear

This iterated logic requires common knowledge of rationality.

Iterated elimination: Step 1

Consider the game

	L	M	R
T	$(3,3)$	$(2,2)$	$(4,1)$
M	$(2,2)$	$(1,1)$	$(3,0)$
B	$(1,0)$	$(0,3)$	$(2,2)$

For player 1, compare T and B:

$$3 > 1, \qquad 2 > 0, \qquad 4 > 2.$$

So B is strictly dominated by T.

We may therefore eliminate B and continue with the reduced game:

	L	M	R
T	$(3,3)$	$(2,2)$	$(4,1)$
M	$(2,2)$	$(1,1)$	$(3,0)$

Iterated elimination: Step 2

In the reduced game

	L	M	R
T	$(3,3)$	$(2,2)$	$(4,1)$
M	$(2,2)$	$(1,1)$	$(3,0)$

compare player 2's strategies L and R:

$$3 > 1 \quad \text{if player 1 chooses } T, \qquad 2 > 0 \quad \text{if player 1 chooses } M.$$

So for player 2, R is strictly dominated by L.

Eliminating R leaves

	L	M
T	$(3,3)$	$(2,2)$
M	$(2,2)$	$(1,1)$

Dominance relations can emerge only *after* earlier irrational strategies have been removed.

Iterated elimination: Step 3

Now consider the further reduced game

	L	M
T	$(3,3)$	$(2,2)$
M	$(2,2)$	$(1,1)$

For player 1:

$$3 > 2, \qquad 2 > 1,$$

so M is strictly dominated by T.

After removing player 1's M, player 2 compares L and M only against T:

$$3 > 2.$$

Hence M is strictly dominated by L.

The unique surviving strategy profile is therefore

$$(T, L).$$

The game is solved by repeated elimination of strictly dominated strategies.

Iterated elimination of strictly dominated strategies

Procedure

1. Identify any strictly dominated strategies.
2. Eliminate them from the game.
3. Recompute dominance relations in the reduced game.
4. Continue until no strictly dominated strategy remains.

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A game is dominance-solvable if this process leaves exactly one strategy profile.

Why this concept is useful

- it extracts predictions from rationality alone;
- it sometimes yields a unique outcome without solving best-response systems;
- it shows clearly which strategies are never worth considering.

But many economically important games are *not* dominance-solvable.

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Eliminating a player's own strictly dominated strategy uses that player's rationality.

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- the opponents know that others are rational,
- the opponents know that others know that others are rational,

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- and so on.

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- the opponents are rational,
- the opponents know that others are rational,
- the opponents know that others know that others are rational,
- and so on.

So the justification for iterated elimination is not just: "I am rational."
It is closer to

"Rationality is common knowledge."

This is the first place where higher-order reasoning enters the lecture.

Prisoner's Dilemma as a dominance-solvable game

Recall the Prisoner's Dilemma:

	C	D
C	$(3,3)$	$(0,4)$
D	$(4,0)$	$(1,1)$

For player 1:

$$4 > 3 \quad \text{if player 2 chooses } C, \qquad 1 > 0 \quad \text{if player 2 chooses } D.$$

So D strictly dominates C.

By symmetry, the same is true for player 2.

Hence repeated elimination leaves the unique prediction

$$(D, D).$$

Economic lesson: individually rational behavior can generate a Pareto-inferior outcome.

Weak dominance

Definition (Weak dominance)

Strategy s_i is weakly dominated by strategy s_i' if

$$u_i(s_i', s_{-i}) \geq u_i(s_i, s_{-i}) \qquad \text{for every } s_{-i} \in S_{-i},$$

and

$$u_i(s_i', s_{-i}) > u_i(s_i, s_{-i}) \qquad \text{for at least one } s_{-i}.$$

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and

$$u_i(s_i', s_{-i}) > u_i(s_i, s_{-i}) \qquad \text{for at least one } s_{-i}.$$

So a weakly dominated strategy is never better and sometimes worse.

This sounds close to strict dominance, but the interpretation is more delicate:

- equality is allowed for some opponent strategies;
- a weakly dominated strategy may still be a best response to some beliefs;
- eliminating it is therefore less innocuous.

Why weak dominance is more delicate

Consider

	L	R
T	$(1,1)$	$(0,0)$
B	$(0,0)$	$(0,0)$

For player 1, B is weakly dominated by T:

$$1 > 0 \text{ against } L, \qquad 0 = 0 \text{ against } R.$$

But note:

- (T, L) is a Nash equilibrium;
- (B, R) is also a Nash equilibrium.

If we eliminate weakly dominated strategy B, the equilibrium (B, R) disappears.

Lesson: iterated elimination of weakly dominated strategies can remove Nash equilibria and is generally order-sensitive.

What dominance can and cannot do

What dominance does well

- it rules out strategies that rational players should never use;
- it gives a first prediction without requiring equilibrium reasoning;
- in some games, it solves the game completely.

What dominance cannot generally do

- it often leaves several strategies alive;
- it does not in general yield a unique outcome;
- weak-dominance reasoning is more fragile than strict-dominance reasoning.

So the next question is unavoidable:

When dominance does not pin down play, what further discipline should a solution concept impose?

Nash equilibrium

Review: Strategic Form Games and Nash Equilibrium

When dominance does not pin down play

Dominance is a useful first filter, but many games are *not* dominance-solvable.

Consider the following game:

	L	R
U	$(2, 2)$	$(0, 0)$
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Consider the following game:

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There is no strictly dominated strategy:

- for player 1, U is better against L, but D is better against R;
- for player 2, L is better against U, but R is better against D.

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Consider the following game:

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There is no strictly dominated strategy:

- for player 1, U is better against L, but D is better against R;
- for player 2, L is better against U, but R is better against D.

So dominance does not tell us what will happen.

New question: if common belief in rationality alone does not select an outcome, what further consistency condition should a prediction satisfy?

Best responses in a payoff matrix

A natural idea is to ask:

Given what the other player does, what is my optimal reply?

In the game

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player 1's best responses are:

- to L: choose U,
- to R: choose D.

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player 1's best responses are:

- to L: choose U,
- to R: choose D.

player 2's best responses are:

- to U: choose L,
- to D: choose R.

So two cells are special:

$$(U, L) \qquad \text{and} \qquad (D, R).$$

Nash equilibrium

Definition

A strategy profile $s^*(s_1^*, \ldots, s_n^*) \in S$ is a Nash equilibrium if, for every player i,

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \qquad \text{for all } s_i \in S_i.$$

So at a Nash equilibrium:

- each player's strategy is optimal *given the others' strategies*;
- no player can gain from a unilateral deviation;
- the prediction is stable against one-player deviations.

Nash equilibrium does *not* say that the outcome is efficient, fair, or globally optimal. It says only that no single player wants to deviate on her own.

What Nash equilibrium says in plain language

Nash equilibrium adds something that dominance alone does not provide.

It requires mutual consistency:

- each player chooses optimally,
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Interpretation: Nash equilibrium is a *self-consistent* prediction of strategic behavior.

Best response

For player i, a strategy $s_i^* \in S_i$ is a best response to opponents' strategy profile s_{-i} if

$$u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i}) \qquad \text{for all } s_i \in S_i.$$

The set of best responses of player i to s_{-i} is often written as

$$BR_i(s_{-i}).$$

This notation is useful because the Nash condition can then be written compactly as

$$s_i^* \in BR_i(s_{-i}^*) \qquad \text{for every } i.$$

To check whether a profile is a Nash equilibrium, we ask:

Is each player's strategy a best response to the others?

Nash equilibrium as mutual best response

A strategy profile s^* is a Nash equilibrium if and only if

$$s_i^* \in BR_i(s_{-i}^*) \qquad \text{for all } i \in N.$$

This gives the practical working definition used in most applications.

Why this formulation is useful

- in matrix games, it tells us where to look: at intersections of best responses;
- in continuous games, it gives a system of best-response equations;
- in empirical work, it clarifies what the benchmark predicts before we study deviations.

Link to dominance: a strictly dominated strategy can never be a best response, but many games with no dominated strategies still have Nash equilibria.

How to find pure-strategy Nash equilibria in a matrix

To find pure-strategy Nash equilibria in a matrix game:

1. For each column, identify player 1's highest payoff.
2. For each row, identify player 2's highest payoff.
3. A cell is a Nash equilibrium if both players' payoffs are best responses there.

Apply this to

	L	R
U	$(2,2)$	$(0,0)$
D	$(0,0)$	$(1,1)$

For player 1:

$$BR_1(L) = \{U\}, \qquad BR_1(R) = \{D\}.$$

For player 2:

$$BR_2(U) = \{L\}, \qquad BR_2(D) = \{R\}.$$

Hence the pure-strategy Nash equilibria are

$$(U, L) \qquad \text{and} \qquad (D, R).$$

What kind of discipline does Nash impose?

It imposes three layers of discipline:

- **Optimization:** each player chooses a best response.
- **Consistency:** each player's choice is optimal given the others' predicted choices.
- **Stability against unilateral deviations:** no player wants to move away on her own.

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But Nash equilibrium still leaves important questions open:

- How do players arrive at the equilibrium?
- Why should equilibrium expectations be correct?
- Which equilibrium is selected if there are several?

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- Which equilibrium is selected if there are several?

This is precisely where behavioral game theory becomes important.

Example: Cournot best responses

Two firms Cournot's oligopoloy

Two firms choosing quantities $q_1, q_2 \geq 0$ simultaneously. Inverse demand is

$$P(Q) = a - bQ, \qquad Q = q_1 + q_2,$$

with $a > c$ and constant marginal cost c. Firm i's profit is

$$\pi_i(q_i, q_j) = q_i(a - b(q_i + q_j) - c).$$

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$$a - c - 2bq_1 - bq_2 = 0,$$

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For firm 1, maximizing π_1 with respect to q_1 gives

$$a - c - 2bq_1 - bq_2 = 0,$$

so the best-response function is

$$BR_1(q_2) = \frac{a - c - bq_2}{2b}.$$

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$$BR_1(q_2) = \frac{a - c - bq_2}{2b}.$$

By symmetry,

$$BR_2(q_1) = \frac{a - c - bq_1}{2b}.$$

Cournot equilibrium

At a Nash equilibrium, both firms choose best responses simultaneously:

$$q_1 = \frac{a - c - bq_2}{2b}, \qquad q_2 = \frac{a - c - bq_1}{2b}.$$

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Substituting gives

$$q^* = \frac{a - c}{3b}.$$

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$$q_1 = \frac{a - c - bq_2}{2b}, \qquad q_2 = \frac{a - c - bq_1}{2b}.$$

In the symmetric equilibrium,

$$q_1^* = q_2^* = q^*.$$

Substituting gives

$$q^* = \frac{a - c}{3b}.$$

Hence total output is

$$Q^* = \frac{2(a - c)}{3b},$$

and the market price is

$$P^* = a - bQ^* = a - \frac{2(a - c)}{3} = \frac{a + 2c}{3}.$$

Economic interpretation: equilibrium output lies between monopoly output and the competitive benchmark.

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Nash equilibrium

- selects profiles of mutual best responses;
- gives a general benchmark even when dominance is silent;
- becomes the central reference point for later behavioral evidence.

Transition: the benchmark is now clear, so we can ask what it explains well and what it leaves open.

Why this benchmark matters

Review: Strategic Form Games and Nash Equilibrium

What standard game theory gives us

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- some strategies can be ruled out immediately,
- in special cases, repeated elimination solves the game completely.

Nash equilibrium adds a general prediction concept:

- each player chooses a best response,
- expectations are mutually consistent,
- no player wants to deviate unilaterally.

What the benchmark does not yet explain

Even when standard game theory is internally precise, important descriptive questions remain open.

It does *not* yet tell us

- whether players correctly understand the game they are facing;
- how many steps of strategic reasoning they actually perform;
- how beliefs about others' play are formed in unfamiliar environments;
- how much observed play reflects noise, confusion, or heterogeneous reasoning;
- why subjects often fail to coordinate on one equilibrium when several are available.

Standard theory gives us a benchmark of correctly specified and internally consistent strategic reasoning.

But a descriptive theory of behavior also needs to explain how people reason in practice.

Why behavioral game theory starts from this benchmark

Behavioral game theory does not replace standard game theory. It starts from it and asks where observed behavior departs from it.

That is only meaningful if the benchmark is already clear.

Typical questions in the next lectures will be:

- Do subjects best respond to correct beliefs?
- How deep does strategic reasoning go in one-shot games?
- Are deviations from equilibrium noisy, or systematically structured?
- Which behavioral models improve prediction most?

So later lectures do not abandon the benchmark. They ask *which part of the benchmark fails, why it fails, and how to model that failure*.

For the broader perspective, see Crawford, Costa-Gomes & Iriberri (2013); Camerer (2003).

Bridge to the next lecture

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When subjects face a strategic problem for the first time, do they reason all the way to equilibrium?

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When subjects face a strategic problem for the first time, do they reason all the way to equilibrium?

Guessing games are a natural testing ground because they make strategic depth visible:

- equilibrium reasoning has a sharp prediction;
- limited depth of reasoning generates systematic deviations;
- those deviations can be modeled in a disciplined way.

Main takeaways

1. Strategic form is the language of the model.
   - A game in strategic form specifies players, strategy sets, and payoff functions.
   - It gives a complete representation of strategic interaction, but not yet a prediction.
2. Dominance is the first implication of rationality.
   - Strictly dominated strategies should never be chosen by rational players.
   - Iterated elimination can sometimes solve a game completely, but not always.
3. Nash equilibrium adds mutual consistency.
   - Each player best responds to the others.
   - No player wants to deviate unilaterally.
4. This benchmark is indispensable, but incomplete.
   - It is the standard reference point for later behavioral evidence.
   - It does not yet explain how equilibrium reasoning is reached in practice.

Literature

Review: Strategic Form Games and Nash Equilibrium

References for : benchmark theory

- Nash, J. F. (1950): "Equilibrium Points in N-Person Games." *Proceedings of the National Academy of Sciences* 36(1), 48–49.
- Nash, J. (1951): "Non-Cooperative Games." *Annals of Mathematics* 54(2), 286–295.
- Cournot, A. A. (1838): *Recherches sur les principes mathématiques de la théorie des richesses*.
- Gibbons, R. (1992): *A Primer in Game Theory*. Harvester Wheatsheaf.
- Osborne, M. J. and Rubinstein, A. (1994): *A Course in Game Theory*. MIT Press.

Role in this lecture:

- Nash provides the benchmark solution concept.
- Cournot provides the canonical continuous-strategy application.
- Gibbons and Osborne–Rubinstein are useful review references for the formal material.

References for : behavioral perspective

- Camerer, C. F. (2003): *Behavioral Game Theory: Experiments in Strategic Interaction*. Princeton University Press.
- Crawford, V. P., Costa-Gomes, M. A., and Iriberri, N. (2013): "Structural Models of Nonequilibrium Strategic Thinking: Theory, Evidence, and Applications." *Journal of Economic Literature* 51(1), 5–62.
- Koppel, L., Andersson, D., Johannesson, M., Strømland, E., and Tinghög, G. (2025): "Comprehension in economic games." *Journal of Economic Behavior & Organization* 234, 107039.
- Huang, S. and Golman, R. (2025): "The collective wisdom of behavioral game theory." *Economic Theory* 79(1), 341–356.

Why these references matter here:

- Camerer and Crawford et al. connect the benchmark to later behavioral modeling.
- Koppel et al. motivate why comprehension cannot simply be taken for granted.
- Huang and Golman motivate comparing several behavioral models rather than looking for one universal alternative.

Repetition questions

Review: Strategic Form Games and Nash Equilibrium

Repetition questions I: strategic form

1. What are the three basic ingredients of a game in strategic form?
Sketch: the set of players N, the strategy sets $(S_i)_{i \in N}$, and the payoff functions $(u_i)_{i \in N}$ defined on strategy profiles.

2. What is a strategy profile?
Sketch: a vector $s = (s_1, \ldots, s_n)$ specifying one strategy for each player.

3. Why is strategic form a representation rather than already a prediction?
Sketch: it tells us which choices are available and what payoffs follow from each profile, but it does not yet select which profile should occur.

4. Why is a strategy not always the same thing as a single action?
Sketch: in general, a strategy is a complete contingent plan; in sequential settings it specifies what to do after every possible history.

Repetition questions II: dominance

5. When is a strategy strictly dominated?
Sketch: when another strategy gives a strictly higher payoff against every possible strategy profile of the opponents.

6. Why should a rational player never choose a strictly dominated strategy?
Sketch: because it is never optimal, regardless of beliefs about what others will do.

7. What is iterated elimination of strictly dominated strategies?
Sketch: repeatedly remove strictly dominated strategies, recompute the reduced game, and continue until no further strict dominance remains.

8. What does it mean for a game to be dominance-solvable?
Sketch: iterated elimination of strictly dominated strategies leaves exactly one strategy profile.

Repetition questions III: higher-order reasoning and weak dominance

9. Why does iterated elimination require more than own rationality?
Sketch: later rounds rely on players assuming that others are rational, that others know this, and so on; the standard justification is common knowledge of rationality.

10. Why is the Prisoner's Dilemma dominance-solvable?
Sketch: for each player, defect strictly dominates cooperate, so repeated elimination yields the unique prediction (D, D).

11. When is a strategy weakly dominated?
Sketch: when another strategy gives at least as high a payoff against all opponent strategies and a strictly higher payoff against at least one.

12. Why is weak dominance more delicate than strict dominance?
Sketch: a weakly dominated strategy can still be a best response to some beliefs; eliminating weakly dominated strategies may remove Nash equilibria and can depend on the order of elimination.

Repetition questions IV: Nash equilibrium

13. State the definition of Nash equilibrium.
Sketch: a strategy profile s^* is a Nash equilibrium if each player's strategy s_i^* maximizes payoff given the opponents' strategies s_{-i}^*.

14. What is a best response?
Sketch: a strategy is a best response to opponents' strategies if it yields at least as high a payoff as every alternative strategy.

15. Why can strictly dominated strategies never be part of a Nash equilibrium?
Sketch: because a strictly dominated strategy is never a best response, and equilibrium requires each player's strategy to be a best response.

16. What is the practical matrix method for finding pure-strategy Nash equilibria?
Sketch: mark player 1's best replies in each column and player 2's best replies in each row; intersections of best replies are Nash equilibria.

Repetition questions V: interpretation and application

17. What does Nash equilibrium add that dominance alone does not provide?
Sketch: Nash imposes mutual consistency of choices and beliefs; it gives a prediction even when dominance does not single out an outcome.

18. Why does Nash equilibrium not imply efficiency or fairness?
Sketch: it only rules out profitable unilateral deviations; it does not require the outcome to maximize joint surplus or satisfy any normative criterion.

19. What is the core intuition of the Cournot equilibrium?
Sketch: each firm chooses a quantity that is optimal given the rival's quantity; equilibrium is the intersection of the firms' best-response functions.

20. Why is essential for the rest of the course?
Sketch: later behavioral results are only meaningful relative to a clear benchmark of strategic form, dominance, best responses, and Nash equilibrium.

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- Nash equilibrium = mutual best responses
- Finding (pure) Nash eq using payoff matrix (underline best responses)