General patterns in strategic behavior

Summer 25

Niels Boissonnet, Bielefeld

Discussion

From last lecture to today

We have studied the beauty contest from the context of two models:

From last lecture to today

We have studied the beauty contest from the context of two models:

- QRE: players respond smoothly to exp. payoffs and beliefs as fixed point.

From last lecture to today

We have studied the beauty contest from the context of two models:

- QRE: players respond smoothly to exp. payoffs and beliefs as fixed point.
- NI: players reason through many layers, deeper layers become less precise.

Today we ask whether the same pattern survives in richer normal-form games.

Once we leave the beauty-contest environment, the key issue

From last lecture to today

We have studied the beauty contest from the context of two models:

- QRE: players respond smoothly to exp. payoffs and beliefs as fixed point.
- NI: players reason through many layers, deeper layers become less precise.

Today we ask whether the same pattern survives in richer normal-form games.

Once we leave the beauty-contest environment, the key issue

- is no longer only *how many steps* subjects reason,

From last lecture to today

We have studied the beauty contest from the context of two models:

- QRE: players respond smoothly to exp. payoffs and beliefs as fixed point.
- NI: players reason through many layers, deeper layers become less precise.

Today we ask whether the same pattern survives in richer normal-form games.

Once we leave the beauty-contest environment, the key issue

- is no longer only *how many steps* subjects reason,
- but also *when simple one-step reasoning is enough*,

From last lecture to today

We have studied the beauty contest from the context of two models:

- QRE: players respond smoothly to exp. payoffs and beliefs as fixed point.
- NI: players reason through many layers, deeper layers become less precise.

Today we ask whether the same pattern survives in richer normal-form games.

Once we leave the beauty-contest environment, the key issue

- is no longer only *how many steps* subjects reason,
- but also *when simple one-step reasoning is enough*,
- *when a smoother equilibrium logic is the better description*.

Why this lecture is needed

This matters because in some games

Why this lecture is needed

This matters because in some games

- Subjects seem to behave as if they take only one simple strategic step.

Why this lecture is needed

This matters because in some games

- Subjects seem to behave as if they take only one simple strategic step.
- Or aggregate play is too smooth and payoff-sensitive for a literal level-1 reading.

Why this lecture is needed

This matters because in some games

- Subjects seem to behave as if they take only one simple strategic step.
- Or aggregate play is too smooth and payoff-sensitive for a literal level-1 reading.
- So we need to understand *when level-k is the right approximation* and *when logit-equilibrium reasoning is the better benchmark*.

Two classic results make this especially sharp:

Why this lecture is needed

This matters because in some games

- Subjects seem to behave as if they take only one simple strategic step.
- Or aggregate play is too smooth and payoff-sensitive for a literal level-1 reading.
- So we need to understand *when level-k is the right approximation* and *when logit-equilibrium reasoning is the better benchmark*.

Two classic results make this especially sharp:

- Weizsäcker (2003): players react more sharply to their *own* incentives than to the rationality they attribute to others.

Why this lecture is needed

This matters because in some games

- Subjects seem to behave as if they take only one simple strategic step.
- Or aggregate play is too smooth and payoff-sensitive for a literal level-1 reading.
- So we need to understand *when level-k is the right approximation* and *when logit-equilibrium reasoning is the better benchmark*.

Two classic results make this especially sharp:

- Weizsäcker (2003): players react more sharply to their *own* incentives than to the rationality they attribute to others.
- Costa-Gomes and Weizsäcker (2008): subjects' *stated beliefs* often do not line up with the actions they actually choose.

Two organizing questions

Q1: When level-k, when QRE?

Which features of a game make behavior look essentially like level-1 choice, and which features make it look more like logit-equilibrium / QRE-type play?

Q2: Do level-1 players really believe opponents randomize uniformly?

If actions are well described by level-1 behavior, should we take the underlying level-0 belief literally — or is level-1 better seen as a useful reduced-form description of how subjects choose?

These questions fit together naturally:

Two organizing questions

Q1: When level-k, when QRE?

Which features of a game make behavior look essentially like level-1 choice, and which features make it look more like logit-equilibrium / QRE-type play?

Q2: Do level-1 players really believe opponents randomize uniformly?

If actions are well described by level-1 behavior, should we take the underlying level-0 belief literally — or is level-1 better seen as a useful reduced-form description of how subjects choose?

These questions fit together naturally:

- Dominated actions often create a simple one-step logic.

Two organizing questions

Q1: When level-k, when QRE?

Which features of a game make behavior look essentially like level-1 choice, and which features make it look more like logit-equilibrium / QRE-type play?

Q2: Do level-1 players really believe opponents randomize uniformly?

If actions are well described by level-1 behavior, should we take the underlying level-0 belief literally — or is level-1 better seen as a useful reduced-form description of how subjects choose?

These questions fit together naturally:

- Dominated actions often create a simple one-step logic.
- Games without dominated actions leave less scope for such shortcuts.

Two organizing questions

Q1: When level-k, when QRE?

Which features of a game make behavior look essentially like level-1 choice, and which features make it look more like logit-equilibrium / QRE-type play?

Q2: Do level-1 players really believe opponents randomize uniformly?

If actions are well described by level-1 behavior, should we take the underlying level-0 belief literally — or is level-1 better seen as a useful reduced-form description of how subjects choose?

These questions fit together naturally:

- Dominated actions often create a simple one-step logic.
- Games without dominated actions leave less scope for such shortcuts.
- Belief elicitation test whether the level-1 interpretation should be literal on the belief side.

Learning goals and route through the lecture

By the end of this lecture, you should be able to:

Learning goals and route through the lecture

By the end of this lecture, you should be able to:

- explain why dominated actions tend to push behavior toward a level-1 interpretation;

Learning goals and route through the lecture

By the end of this lecture, you should be able to:

- explain why dominated actions tend to push behavior toward a level-1 interpretation;
- explain why, in games without dominated actions, aggregate play is often better described by logit equilibrium / QRE-type response;

Learning goals and route through the lecture

By the end of this lecture, you should be able to:

- explain why dominated actions tend to push behavior toward a level-1 interpretation;
- explain why, in games without dominated actions, aggregate play is often better described by logit equilibrium / QRE-type response;
- interpret asymmetric logit equilibrium (ALE) as a family connecting level-1, QRE, and Nash equilibrium;

Learning goals and route through the lecture

By the end of this lecture, you should be able to:

- explain why dominated actions tend to push behavior toward a level-1 interpretation;
- explain why, in games without dominated actions, aggregate play is often better described by logit equilibrium / QRE-type response;
- interpret asymmetric logit equilibrium (ALE) as a family connecting level-1, QRE, and Nash equilibrium;
- explain why stated beliefs need not coincide with the literal level-0 beliefs that would justify textbook level-1 choice.

Route through the lecture

Learning goals and route through the lecture

By the end of this lecture, you should be able to:

- explain why dominated actions tend to push behavior toward a level-1 interpretation;
- explain why, in games without dominated actions, aggregate play is often better described by logit equilibrium / QRE-type response;
- interpret asymmetric logit equilibrium (ALE) as a family connecting level-1, QRE, and Nash equilibrium;
- explain why stated beliefs need not coincide with the literal level-0 beliefs that would justify textbook level-1 choice.

Route through the lecture

1. ALE as a bridge between one-step reasoning and smooth equilibrium response

Learning goals and route through the lecture

By the end of this lecture, you should be able to:

- explain why dominated actions tend to push behavior toward a level-1 interpretation;
- explain why, in games without dominated actions, aggregate play is often better described by logit equilibrium / QRE-type response;
- interpret asymmetric logit equilibrium (ALE) as a family connecting level-1, QRE, and Nash equilibrium;
- explain why stated beliefs need not coincide with the literal level-0 beliefs that would justify textbook level-1 choice.

Route through the lecture

1. ALE as a bridge between one-step reasoning and smooth equilibrium response
2. Evidence by game structure: dominated actions versus no dominated actions

Learning goals and route through the lecture

By the end of this lecture, you should be able to:

- explain why dominated actions tend to push behavior toward a level-1 interpretation;
- explain why, in games without dominated actions, aggregate play is often better described by logit equilibrium / QRE-type response;
- interpret asymmetric logit equilibrium (ALE) as a family connecting level-1, QRE, and Nash equilibrium;
- explain why stated beliefs need not coincide with the literal level-0 beliefs that would justify textbook level-1 choice.

Route through the lecture

1. ALE as a bridge between one-step reasoning and smooth equilibrium response
2. Evidence by game structure: dominated actions versus no dominated actions
3. Belief elicitation and the question whether level-1 beliefs are literal

Learning goals and route through the lecture

By the end of this lecture, you should be able to:

- explain why dominated actions tend to push behavior toward a level-1 interpretation;
- explain why, in games without dominated actions, aggregate play is often better described by logit equilibrium / QRE-type response;
- interpret asymmetric logit equilibrium (ALE) as a family connecting level-1, QRE, and Nash equilibrium;
- explain why stated beliefs need not coincide with the literal level-0 beliefs that would justify textbook level-1 choice.

Route through the lecture

1. ALE as a bridge between one-step reasoning and smooth equilibrium response
2. Evidence by game structure: dominated actions versus no dominated actions
3. Belief elicitation and the question whether level-1 beliefs are literal
4. What this implies for behavioral modeling in normal-form games

Asymmetric Logit Equilibrium

General patterns in strategic behavior

Why start with ALE?

In the BT game: observed behavior looks either like simple one-step reasoning or like smooth equilibrium response, but not like a single literal finite level-k model applied uniformly across subjects and games.

Why start with ALE?

In the BT game: observed behavior looks either like simple one-step reasoning or like smooth equilibrium response, but not like a single literal finite level-k model applied uniformly across subjects and games.

Normal-form games sharpen this question:

Why start with ALE?

In the BT game: observed behavior looks either like simple one-step reasoning or like smooth equilibrium response, but not like a single literal finite level-k model applied uniformly across subjects and games.

Normal-form games sharpen this question:

- If a game contains a dominated action, subjects may only need one step.

Why start with ALE?

In the BT game: observed behavior looks either like simple one-step reasoning or like smooth equilibrium response, but not like a single literal finite level-k model applied uniformly across subjects and games.

Normal-form games sharpen this question:

- If a game contains a dominated action, subjects may only need one step.
- If not, then no shortcut, so behavior may look more like QRE.

Why start with ALE?

In the BT game: observed behavior looks either like simple one-step reasoning or like smooth equilibrium response, but not like a single literal finite level-k model applied uniformly across subjects and games.

Normal-form games sharpen this question:

- If a game contains a dominated action, subjects may only need one step.
- If not, then no shortcut, so behavior may look more like QRE.

$\Longrightarrow$ We thus want a model family that *connects* these cases rather than choosing one benchmark in advance.

Asymmetric logit equilibrium (ALE)

ALE is useful because it nests level-1, logit equilibrium, and Nash equilibrium in one continuum.

Asymmetric logit equilibrium (Weizsäcker, 2003)

Asymmetric logit equilibrium (ALE)

Let $r_i(\pi_j)$ denote player i's logit response to strategy π_j with own precision λ_i. Let $\tilde{r}_j(\pi_i)$ denote player i's perceived response of player j to π_i, with perceived precision $\tilde{\lambda}_j$. Strategy π_i is an ALE strategy if

$$\pi_i = r_i(\tilde{r}_j(\pi_i)) \,.$$

Asymmetric logit equilibrium (Weizsäcker, 2003)

Asymmetric logit equilibrium (ALE)

Let $r_i(\pi_j)$ denote player i's logit response to strategy π_j with own precision λ_i. Let $\tilde{r}_j(\pi_i)$ denote player i's perceived response of player j to π_i, with perceived precision $\tilde{\lambda}_j$. Strategy π_i is an ALE strategy if

$$\pi_i = r_i(\tilde{r}_j(\pi_i))\,.$$

- Level-1: $\lambda_i = \infty,\ \tilde{\lambda}_j = 0$

Asymmetric logit equilibrium (Weizsäcker, 2003)

Asymmetric logit equilibrium (ALE)

Let $r_i(\pi_j)$ denote player i's logit response to strategy π_j with own precision λ_i. Let $\tilde{r}_j(\pi_i)$ denote player i's perceived response of player j to π_i, with perceived precision $\tilde{\lambda}_j$. Strategy π_i is an ALE strategy if

$$\pi_i = r_i(\tilde{r}_j(\pi_i))\,.$$

- Level-1: $\lambda_i = \infty,\ \tilde{\lambda}_j = 0$
- Logit equilibrium / QRE: $\lambda_i = \tilde{\lambda}_j$

Asymmetric logit equilibrium (Weizsäcker, 2003)

Asymmetric logit equilibrium (ALE)

Let $r_i(\pi_j)$ denote player i's logit response to strategy π_j with own precision λ_i. Let $\tilde{r}_j(\pi_i)$ denote player i's perceived response of player j to π_i, with perceived precision $\tilde{\lambda}_j$. Strategy π_i is an ALE strategy if

$$\pi_i = r_i(\tilde{r}_j(\pi_i)) \,.$$

- Level-1: $\lambda_i = \infty,\ \tilde{\lambda}_j = 0$
- Logit equilibrium / QRE: $\lambda_i = \tilde{\lambda}_j$
- Nash equilibrium: $\lambda_i = \tilde{\lambda}_j = \infty$

ALE is a fixed point to be computed by function iteration, just like QRE.

What ALE means behaviorally

Own precision λ_i

What ALE means behaviorally

Own precision λ_i

- How sharply I respond to expected payoff differences

What ALE means behaviorally

Own precision λ_i

- How sharply I respond to expected payoff differences
- High λ_i: choices concentrate on better replies

What ALE means behaviorally

Own precision λ_i

- How sharply I respond to expected payoff differences
- High λ_i: choices concentrate on better replies
- Low λ_i: behavior is more diffuse

Perceived opponent precision $\tilde{\lambda}_j$

What ALE means behaviorally

Own precision λ_i

- How sharply I respond to expected payoff differences
- High λ_i: choices concentrate on better replies
- Low λ_i: behavior is more diffuse

Perceived opponent precision $\tilde{\lambda}_j$

- How sharply I think *you* respond to payoffs

What ALE means behaviorally

Own precision λ_i

- How sharply I respond to expected payoff differences
- High λ_i: choices concentrate on better replies
- Low λ_i: behavior is more diffuse

Perceived opponent precision $\tilde{\lambda}_j$

- How sharply I think *you* respond to payoffs
- High $\tilde{\lambda}_j$: I expect you to optimize strongly

What ALE means behaviorally

Own precision λ_i

- How sharply I respond to expected payoff differences
- High λ_i: choices concentrate on better replies
- Low λ_i: behavior is more diffuse

Perceived opponent precision $\tilde{\lambda}_j$

- How sharply I think *you* respond to payoffs
- High $\tilde{\lambda}_j$: I expect you to optimize strongly
- Low $\tilde{\lambda}_j$: I treat your behavior as relatively unsophisticated

Key interpretation

If $\lambda_i > \tilde{\lambda}_j$, then I behave as if *I optimize more than I think you do*.

This is exactly the asymmetry we want to study today.

Data and estimation strategy

Data Re-analysis of 22 normal-form games (2×2, 2×3, 3×3) from Stahl and Wilson (1994, 1995) and Costa-Gomes et al. (2001).

Econometric model

Data and estimation strategy

Data Re-analysis of 22 normal-form games (2×2, 2×3, 3×3) from Stahl and Wilson (1994, 1995) and Costa-Gomes et al. (2001).

Econometric model

- Mixed logit specification

Data and estimation strategy

Data Re-analysis of 22 normal-form games (2×2, 2×3, 3×3) from Stahl and Wilson (1994, 1995) and Costa-Gomes et al. (2001).

Econometric model

- Mixed logit specification
- Own precision λ_i and perceived opponent precision $\tilde{\lambda}_j$ are drawn independently from Gamma distributions

Comparison benchmark: ANNE

Data and estimation strategy

Data Re-analysis of 22 normal-form games (2×2, 2×3, 3×3) from Stahl and Wilson (1994, 1995) and Costa-Gomes et al. (2001).

Econometric model

- Mixed logit specification
- Own precision λ_i and perceived opponent precision $\tilde{\lambda}_j$ are drawn independently from Gamma distributions

Comparison benchmark: ANNE

- Asymmetric Noisy Nash Equilibrium (ANNE) replaces random-utility noise by trembles

Data and estimation strategy

Data Re-analysis of 22 normal-form games (2×2, 2×3, 3×3) from Stahl and Wilson (1994, 1995) and Costa-Gomes et al. (2001).

Econometric model

- Mixed logit specification
- Own precision λ_i and perceived opponent precision $\tilde{\lambda}_j$ are drawn independently from Gamma distributions

Comparison benchmark: ANNE

- Asymmetric Noisy Nash Equilibrium (ANNE) replaces random-utility noise by trembles
- Tremble probabilities $(\epsilon_i, \tilde{\epsilon}_j)$ are drawn from independent Beta distributions

Why this comparison matters

If ALE fits better than ANNE, this points toward payoff-sensitive noisy response rather than mere random mistakes.

Baseline evidence: players simplify opponents

Table 2
ML estimates of distribution parameters, using pooled experimental data

ALE parameters		ANNE parameters	
ρ	0.35	a	2.28
K	2.49	b	5.28
$\tilde{\rho}$	0.15	$\tilde{a}$	0.77
$\widetilde{K}$	0.58	$\tilde{b}$	0.55
l^*	−907.154	l^*	−930.039

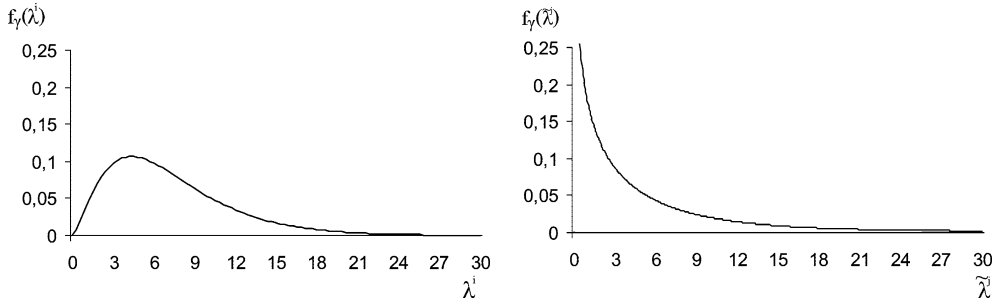


Fig. 3. Estimated densities of ALE parameters using pooled data. (a) Depicts the estimated density $f_\gamma(\lambda^i \mid \rho, K)$; (b) the estimated density $f_\gamma(\tilde{\lambda}^j \mid \tilde{\rho}, \widetilde{K})$.

Main result

Average own precision is substantially larger than average perceived opponent precision (roughly $\lambda \approx 4.1$ versus $\tilde{\lambda} \approx 2.2$). So symmetric QRE is too restrictive.

The asymmetry is not constant across games

Separate estimations of the ALE strategy model for the data from Stahl and Wilson (1994, 1995) and Costa-Gomes et al. (2001)

Data set	SW 1994	SW 1995	CGCB 2001
ρ	6.55	7.21	0.23
K	171.24	53.06	1.79
$\tilde{\rho}$	0.43	0.07	2.08
$\widetilde{K}$	0.75	0.82	4.60
l^*	-140.468	-151.128	-571.853
$E[\lambda^i]$	26.13	7.36	7.74
$E[\tilde{\lambda}^j]$	1.75	11.33	2.21
sig$(E[\lambda^i] = E[\tilde{\lambda}^j])$	3.322×10^{-10}	0.062	2.078×10^{-7}
sig$(\rho = \tilde{\rho}, K = \widetilde{K})$	8.623×10^{-13}	0.019	8.620×10^{-8}
sig$((\bar{\lambda}^i, \bar{\tilde{\lambda}}^j)$ fix over $i)$	4.706×10^{-11}	3.443×10^{-6}	2.258×10^{-25}
sig$(\bar{\lambda}^i = \bar{\tilde{\lambda}}^j = \lambda$, fix over $i)$	4.720×10^{-19}	1.159×10^{-5}	7.509×10^{-31}
# of model rejections at $p = 5\%$	0 (out of 5)	1 (out of 4)	8 (out of 28)
# of subjects with $\lambda^i > \tilde{\lambda}^j$	29 (16 sig.)	25 (9 sig.)	61 (26 sig.)
# of subjects with $\lambda^i \leqslant \tilde{\lambda}^j$	11 (0 sig.)	23 (0 sig.)	11 (0 sig.)

The asymmetry is not constant across games

Separate estimations of the ALE strategy model for the data from Stahl and Wilson (1994, 1995) and Costa-Gomes et al. (2001)

Data set	SW 1994	SW 1995	CGCB 2001
ρ	6.55	7.21	0.23
K	171.24	53.06	1.79
$\tilde{\rho}$	0.43	0.07	2.08
$\widetilde{K}$	0.75	0.82	4.60
l^*	-140.468	-151.128	-571.853
$E[\lambda^i]$	26.13	7.36	7.74
$E[\tilde{\lambda}^j]$	1.75	11.33	2.21
sig$(E[\lambda^i] = E[\tilde{\lambda}^j])$	3.322×10^{-10}	0.062	2.078×10^{-7}
sig$(\rho = \tilde{\rho}, K = \widetilde{K})$	8.623×10^{-13}	0.019	8.620×10^{-8}
sig$((\bar{\lambda}^i, \bar{\tilde{\lambda}}^j)$ fix over $i)$	4.706×10^{-11}	3.443×10^{-6}	2.258×10^{-25}
sig$(\bar{\lambda}^i = \bar{\tilde{\lambda}}^j = \lambda$, fix over $i)$	4.720×10^{-19}	1.159×10^{-5}	7.509×10^{-31}
# of model rejections at $p = 5\%$	0 (out of 5)	1 (out of 4)	8 (out of 28)
# of subjects with $\lambda^i > \tilde{\lambda}^j$	29 (16 sig.)	25 (9 sig.)	61 (26 sig.)
# of subjects with $\lambda^i \leqslant \tilde{\lambda}^j$	11 (0 sig.)	23 (0 sig.)	11 (0 sig.)

- The degree of asymmetry varies strongly across the underlying datasets.

The asymmetry is not constant across games

Separate estimations of the ALE strategy model for the data from Stahl and Wilson (1994, 1995) and Costa-Gomes et al. (2001)

Data set	SW 1994	SW 1995	CGCB 2001
ρ	6.55	7.21	0.23
K	171.24	53.06	1.79
$\tilde{\rho}$	0.43	0.07	2.08
$\widetilde{K}$	0.75	0.82	4.60
l^*	-140.468	-151.128	-571.853
$E[\lambda^i]$	26.13	7.36	7.74
$E[\tilde{\lambda}^j]$	1.75	11.33	2.21
sig$(E[\lambda^i] = E[\tilde{\lambda}^j])$	3.322×10^{-10}	0.062	2.078×10^{-7}
sig$(\rho = \tilde{\rho}, K = \widetilde{K})$	8.623×10^{-13}	0.019	8.620×10^{-8}
sig$((\bar{\lambda}^i, \bar{\tilde{\lambda}}^j)$ fix over $i)$	4.706×10^{-11}	3.443×10^{-6}	2.258×10^{-25}
sig$(\bar{\lambda}^i = \bar{\tilde{\lambda}}^j = \lambda$, fix over $i)$	4.720×10^{-19}	1.159×10^{-5}	7.509×10^{-31}
# of model rejections at $p = 5\%$	0 (out of 5)	1 (out of 4)	8 (out of 28)
# of subjects with $\lambda^i > \tilde{\lambda}^j$	29 (16 sig.)	25 (9 sig.)	61 (26 sig.)
# of subjects with $\lambda^i \leqslant \tilde{\lambda}^j$	11 (0 sig.)	23 (0 sig.)	11 (0 sig.)

- The degree of asymmetry varies strongly across the underlying datasets.
- So not just: "players always simplify opponents by the same amount."

Dominated actions, essentially level-1.

Data set	DD	NDD
ρ	0.09	38.70
K	1.27	171.03
$\tilde{\rho}$	1.43	8.04
$\widetilde{K}$	3.03	35.94
l^*	-583.582	-326.213
$E[\lambda^i]$	13.14	4.42
$E[\tilde{\lambda}^j]$	2.12	4.47
sig$(E[\lambda^i] = E[\tilde{\lambda}^j])$	5.045×10^{-9}	0.942
sig$(\rho = \tilde{\rho}, K = \widetilde{K})$	4.647×10^{-12}	0.976
sig$((\bar{\lambda}^i, \bar{\tilde{\lambda}}^j)$ fix over $i)$	2.924×10^{-29}	0.976
sig$(\bar{\lambda}^i = \bar{\tilde{\lambda}}^j = \lambda$, fix over $i)$	2.073×10^{-36}	0.997
# of model rejections at $p = 5\%$	7 (out of 28)	4 (out of 9)
# of cases with $\lambda^i > \tilde{\lambda}^j$	106 (29 sig.)	79 (9 sig.)
# of cases with $\lambda^i \leqslant \tilde{\lambda}^j$	54 (0 sig.)	81 (0 sig.)

Dominated actions, essentially level-1.

Data set	DD	NDD
ρ	0.09	38.70
K	1.27	171.03
$\tilde{\rho}$	1.43	8.04
$\widetilde{K}$	3.03	35.94
l^*	-583.582	-326.213
$E[\lambda^i]$	13.14	4.42
$E[\tilde{\lambda}^j]$	2.12	4.47
sig$(E[\lambda^i] = E[\tilde{\lambda}^j])$	5.045×10^{-9}	0.942
sig$(\rho = \tilde{\rho}, K = \widetilde{K})$	4.647×10^{-12}	0.976
sig$((\bar{\lambda}^i, \bar{\tilde{\lambda}}^j)$ fix over $i)$	2.924×10^{-29}	0.976
sig$(\bar{\lambda}^i = \bar{\tilde{\lambda}}^j = \lambda$, fix over $i)$	2.073×10^{-36}	0.997
# of model rejections at $p = 5\%$	7 (out of 28)	4 (out of 9)
# of cases with $\lambda^i > \tilde{\lambda}^j$	106 (29 sig.)	79 (9 sig.)
# of cases with $\lambda^i \leqslant \tilde{\lambda}^j$	54 (0 sig.)	81 (0 sig.)

- The estimated asymmetry becomes much stronger when the game contains a dominated strategy for one of the players.

Dominated actions, essentially level-1.

Data set	DD	NDD
ρ	0.09	38.70
K	1.27	171.03
$\tilde{\rho}$	1.43	8.04
$\widetilde{K}$	3.03	35.94
l^*	-583.582	-326.213
$E[\lambda^i]$	13.14	4.42
$E[\tilde{\lambda}^j]$	2.12	4.47
sig$(E[\lambda^i] = E[\tilde{\lambda}^j])$	5.045×10^{-9}	0.942
sig$(\rho = \tilde{\rho}, K = \widetilde{K})$	4.647×10^{-12}	0.976
sig$((\bar{\lambda}^i, \bar{\tilde{\lambda}}^j)$ fix over $i)$	2.924×10^{-29}	0.976
sig$(\bar{\lambda}^i = \bar{\tilde{\lambda}}^j = \lambda$, fix over $i)$	2.073×10^{-36}	0.997
# of model rejections at $p = 5\%$	7 (out of 28)	4 (out of 9)
# of cases with $\lambda^i > \tilde{\lambda}^j$	106 (29 sig.)	79 (9 sig.)
# of cases with $\lambda^i \leqslant \tilde{\lambda}^j$	54 (0 sig.)	81 (0 sig.)

- The estimated asymmetry becomes much stronger when the game contains a dominated strategy for one of the players.
- ALE fits better than ANNE especially in those games.

Dominated actions, essentially level-1.

Data set	DD	NDD
ρ	0.09	38.70
K	1.27	171.03
$\tilde{\rho}$	1.43	8.04
$\widetilde{K}$	3.03	35.94
l^*	-583.582	-326.213
$E[\lambda^i]$	13.14	4.42
$E[\tilde{\lambda}^j]$	2.12	4.47
sig$(E[\lambda^i] = E[\tilde{\lambda}^j])$	5.045×10^{-9}	0.942
sig$(\rho = \tilde{\rho}, K = \widetilde{K})$	4.647×10^{-12}	0.976
sig$((\bar{\lambda}^i, \bar{\tilde{\lambda}}^j)$ fix over $i)$	2.924×10^{-29}	0.976
sig$(\bar{\lambda}^i = \bar{\tilde{\lambda}}^j = \lambda$, fix over $i)$	2.073×10^{-36}	0.997
# of model rejections at $p = 5\%$	7 (out of 28)	4 (out of 9)
# of cases with $\lambda^i > \tilde{\lambda}^j$	106 (29 sig.)	79 (9 sig.)
# of cases with $\lambda^i \leqslant \tilde{\lambda}^j$	54 (0 sig.)	81 (0 sig.)

- The estimated asymmetry becomes much stronger when the game contains a dominated strategy for one of the players.
- ALE fits better than ANNE especially in those games.
- Interpretation: subjects often eliminate *their own* dominated option, but they do not fully project the same reasoning onto the opponent.

Illustr.: G1 versus G8 in Stahl and Wilson (1994)

Game 1

	T'	M'	B'
T	40,40	10,20	70,30
M	20,10	80,80	0,100
B	30,70	100,0	60,60

Illustr.: G1 versus G8 in Stahl and Wilson (1994)

Game 1

	T'	M'	B'
T	40,40	10,20	70,30
M	20,10	80,80	0,100
B	30,70	100,0	60,60

- $B > M$ for both players

Illustr.: G1 versus G8 in Stahl and Wilson (1994)

Game 1

	T'	M'	B'
T	40,40	10,20	70,30
M	20,10	80,80	0,100
B	30,70	100,0	60,60

- $B > M$ for both players

$\Longrightarrow$ Subjects recognize this: rarely choose M

Illustr.: G1 versus G8 in Stahl and Wilson (1994)

Game 1

	T'	M'	B'
T	40,40	10,20	70,30
M	20,10	80,80	0,100
B	30,70	100,0	60,60

- $B > M$ for both players

$\Longrightarrow$ Subjects recognize this: rarely choose M

- But they do not fully infer the same dominance reasoning for the opponent

Illustr.: G1 versus G8 in Stahl and Wilson (1994)

Game 1

	T'	M'	B'
T	40,40	10,20	70,30
M	20,10	80,80	0,100
B	30,70	100,0	60,60

Game 8

	T'	M'	B'
T	10,10	100,0	0,78
M	0,100	60,60	70,40
B	78,0	40,70	20,20

- $B > M$ for both players

$\Longrightarrow$ Subjects recognize this: rarely choose M

- But they do not fully infer the same dominance reasoning for the opponent

$\Longrightarrow$ Effective behavior is therefore close to level-1

Illustr.: G1 versus G8 in Stahl and Wilson (1994)

Game 1

	T'	M'	B'
T	40,40	10,20	70,30
M	20,10	80,80	0,100
B	30,70	100,0	60,60

- $B > M$ for both players

$\Longrightarrow$ Subjects recognize this: rarely choose M

- But they do not fully infer the same dominance reasoning for the opponent

$\Longrightarrow$ Effective behavior is therefore close to level-1

Game 8

	T'	M'	B'
T	10,10	100,0	0,78
M	0,100	60,60	70,40
B	78,0	40,70	20,20

- No dominated strategies

Illustr.: G1 versus G8 in Stahl and Wilson (1994)

Game 1

	T'	M'	B'
T	40,40	10,20	70,30
M	20,10	80,80	0,100
B	30,70	100,0	60,60

- $B > M$ for both players

$\Longrightarrow$ Subjects recognize this: rarely choose M

- But they do not fully infer the same dominance reasoning for the opponent

$\Longrightarrow$ Effective behavior is therefore close to level-1

Game 8

	T'	M'	B'
T	10,10	100,0	0,78
M	0,100	60,60	70,40
B	78,0	40,70	20,20

- No dominated strategies
- Unique mixed equilibrium

Illustr.: G1 versus G8 in Stahl and Wilson (1994)

Game 1

	T'	M'	B'
T	40,40	10,20	70,30
M	20,10	80,80	0,100
B	30,70	100,0	60,60

- $B > M$ for both players

$\Longrightarrow$ Subjects recognize this: rarely choose M

- But they do not fully infer the same dominance reasoning for the opponent

$\Longrightarrow$ Effective behavior is therefore close to level-1

Game 8

	T'	M'	B'
T	10,10	100,0	0,78
M	0,100	60,60	70,40
B	78,0	40,70	20,20

- No dominated strategies
- Unique mixed equilibrium
- Smooth payoff-sensitive response organizes behavior much better

Illustr.: G1 versus G8 in Stahl and Wilson (1994)

Game 1

	T'	M'	B'
T	40,40	10,20	70,30
M	20,10	80,80	0,100
B	30,70	100,0	60,60

- $B > M$ for both players

$\Longrightarrow$ Subjects recognize this: rarely choose M

- But they do not fully infer the same dominance reasoning for the opponent

$\Longrightarrow$ Effective behavior is therefore close to level-1

Game 8

	T'	M'	B'
T	10,10	100,0	0,78
M	0,100	60,60	70,40
B	78,0	40,70	20,20

- No dominated strategies
- Unique mixed equilibrium
- Smooth payoff-sensitive response organizes behavior much better
- Here the data look much more like logit equilibrium

Section 1 takeaway: when level-k, when QRE?

Section 1 takeaway: when level-k, when QRE?

- ALE spans the main benchmarks: level-1, logit equilibrium / QRE, and Nash equilibrium.

Section 1 takeaway: when level-k, when QRE?

- ALE spans the main benchmarks: level-1, logit equilibrium / QRE, and Nash equilibrium.
- The crucial empirical divider is game structure.

Section 1 takeaway: when level-k, when QRE?

- ALE spans the main benchmarks: level-1, logit equilibrium / QRE, and Nash equilibrium.
- The crucial empirical divider is game structure.
- If a game contains dominated actions, observed choice is often essentially level-1.

Section 1 takeaway: when level-k, when QRE?

- ALE spans the main benchmarks: level-1, logit equilibrium / QRE, and Nash equilibrium.
- The crucial empirical divider is game structure.
- If a game contains dominated actions, observed choice is often essentially level-1.
- If a game contains no dominated actions, observed choice is often much closer to logit equilibrium.

Transition

If actions often look level-1, should we conclude that subjects literally believe opponents randomize uniformly? To answer that, we need stated beliefs.

Belief-Action Inconsistency

General patterns in strategic behavior

Why elicit beliefs at all?

Section 1 classified *actions*:

Why elicit beliefs at all?

Section 1 classified *actions*:

- with dominated actions, behavior often looks essentially level-1;

Why elicit beliefs at all?

Section 1 classified *actions*:

- with dominated actions, behavior often looks essentially level-1;
- without dominated actions, behavior is often closer to logit equilibrium.

But this leaves an important open question.

Question for this section

If actions look level-1, should we conclude that subjects literally believe opponents randomize uniformly, as in the textbook level-0 foundation?

Why action data are not enough

Why elicit beliefs at all?

Section 1 classified *actions*:

- with dominated actions, behavior often looks essentially level-1;
- without dominated actions, behavior is often closer to logit equilibrium.

But this leaves an important open question.

Question for this section

If actions look level-1, should we conclude that subjects literally believe opponents randomize uniformly, as in the textbook level-0 foundation?

Why action data are not enough

- From actions alone, beliefs are only *inferred*.

Why elicit beliefs at all?

Section 1 classified *actions*:

- with dominated actions, behavior often looks essentially level-1;
- without dominated actions, behavior is often closer to logit equilibrium.

But this leaves an important open question.

Question for this section

If actions look level-1, should we conclude that subjects literally believe opponents randomize uniformly, as in the textbook level-0 foundation?

Why action data are not enough

- From actions alone, beliefs are only *inferred*.
- So a good fit of level-1 choice does not by itself show that subjects truly hold literal level-0 beliefs.

Why elicit beliefs at all?

Section 1 classified *actions*:

- with dominated actions, behavior often looks essentially level-1;
- without dominated actions, behavior is often closer to logit equilibrium.

But this leaves an important open question.

Question for this section

If actions look level-1, should we conclude that subjects literally believe opponents randomize uniformly, as in the textbook level-0 foundation?

Why action data are not enough

- From actions alone, beliefs are only *inferred*.
- So a good fit of level-1 choice does not by itself show that subjects truly hold literal level-0 beliefs.
- Belief elicitation lets us compare what subjects say they expect with what they actually choose.

Costa-Gomes and Weizsäcker (2008): design

Experimental design

Costa-Gomes and Weizsäcker (2008): design

Experimental design

- 14 one-shot normal-form games

Costa-Gomes and Weizsäcker (2008): design

Experimental design

- 14 one-shot normal-form games
- all games are 3×3

Costa-Gomes and Weizsäcker (2008): design

Experimental design

- 14 one-shot normal-form games
- all games are 3×3
- each game has a unique Nash equilibrium

Costa-Gomes and Weizsäcker (2008): design

Experimental design

- 14 one-shot normal-form games
- all games are 3×3
- each game has a unique Nash equilibrium
- some games are dominance-solvable

Costa-Gomes and Weizsäcker (2008): design

Experimental design

- 14 one-shot normal-form games
- all games are 3×3
- each game has a unique Nash equilibrium
- some games are dominance-solvable
- beliefs are elicited using a proper scoring rule

Why this design is useful here

Costa-Gomes and Weizsäcker (2008): design

Experimental design

- 14 one-shot normal-form games
- all games are 3×3
- each game has a unique Nash equilibrium
- some games are dominance-solvable
- beliefs are elicited using a proper scoring rule

Why this design is useful here

- The larger action space creates room for variation in both actions and beliefs.

Costa-Gomes and Weizsäcker (2008): design

Experimental design

- 14 one-shot normal-form games
- all games are 3×3
- each game has a unique Nash equilibrium
- some games are dominance-solvable
- beliefs are elicited using a proper scoring rule

Why this design is useful here

- The larger action space creates room for variation in both actions and beliefs.
- Dominance-solvable games let us test whether action choice still looks close to level-1.

Costa-Gomes and Weizsäcker (2008): design

Experimental design

- 14 one-shot normal-form games
- all games are 3×3
- each game has a unique Nash equilibrium
- some games are dominance-solvable
- beliefs are elicited using a proper scoring rule

Why this design is useful here

- The larger action space creates room for variation in both actions and beliefs.
- Dominance-solvable games let us test whether action choice still looks close to level-1.
- Direct belief reports let us ask whether those choices are really backed by literal level-0 beliefs.

Costa-Gomes and Weizsäcker (2008): design

#1	*L*	*M*	R
T	78, 73	69, 23	12, 14
M	67, 52	59, 61	78, 53
B	16, 76	65, 87	94, 79

#2	*L*	*M*	*R*
T	21, 67	59, 57	85, 63
M	71, 76	50, 65	74, 14
B	12, 10	51, 76	77, 92

Game #2's pay-offs are obtained by subtracting 2 points from Game #1's pay-offs.

#3	*L*	*M*	R
T	74, 38	78, 71	46, 43
M	96, 12	10, 89	57, 25
B	15, 51	83, 18	69, 62

#4	*L*	*M*	*R*
T	73, 80	20, 85	91, 12
M	45, 48	64, 71	27, 59
B	40, 76	53, 17	14, 98

Game #4's pay-offs are obtained by adding 2 points to Game #3's pay-offs.

#5	*L*	*M*	R
T	78, 49	60, 68	27, 35
M	10, 82	49, 10	98, 38
B	69, 64	42, 39	85, 56

#6	*L*	*M*	*R*
T	39, 99	36, 28	57, 86
M	83, 11	50, 79	65, 70
B	11, 50	69, 61	40, 43

Game #6's pay-offs are obtained by adding 1 point to Game #5's pay-offs.

#7	*L*	*M*	R
T	84, 82	33, 95	12, 73
M	21, 28	39, 37	68, 64
B	70, 39	31, 48	59, 81

#8	*L*	*M*	*R*
T	47, 30	94, 32	36, 38
M	38, 69	81, 83	27, 20
B	80, 58	72, 11	63, 67

Game #8's pay-offs are obtained by subtracting 1 point from Game #7's pay-offs.

#9	*L*	*M*	R
T	57, 58	46, 34	74, 70
M	89, 32	31, 83	12, 41
B	41, 94	16, 37	53, 23

#10	*L*	*M*	*R*
T	60, 59	34, 91	96, 43
M	36, 48	85, 33	39, 18
B	72, 76	43, 14	25, 55

Game #10's pay-offs are obtained by adding 2 points to Game #9's pay-offs.

#11	*L*	*M*	R
T	43, 91	38, 81	92, 64
M	39, 27	79, 68	68, 19
B	69, 10	66, 21	74, 54

#12	*L*	*M*	*R*
T	25, 27	90, 43	38, 60
M	49, 39	53, 73	78, 52
B	64, 85	20, 46	19, 78

#13	*L*	*M*	R
T	83, 40	23, 68	70, 81
M	93, 45	12, 71	29, 41
B	66, 94	56, 76	21, 70

#14	*L*	*M*	*R*
T	82, 61	36, 46	24, 22
M	43, 17	70, 50	40, 87
B	75, 16	49, 75	57, 35

Game #13's pay-offs are obtained by adding 2 points to Game #11's pay-offs; Game #14's pay-offs are obtained by subtracting 3 point from Game # 12's pay-offs

Note: The games are ordered as in Table 1, but with decisions ordered as they appeared to the subjects; the equilibrium is identified by underlining its pay-offs.

FIGURE 1

TABLE 1

Games classified by strategic structure and models' predicted actions

Game	Dominance solvable	Rounds of dominance	Nash	Naïve L 1	L2	D1	Optimistic
#1	Y	2,3	T-L	M-L	T-M	T-L	B-M
#2	Y	3,2	M-L	M-M	T-L	M-L	T-R
#3	Y	2,3	B-R	T-M	B-M	B-M	M-M
#4	Y	3,2	M-M	T-L	T-M	T-M	T-R
#5	Y	2,3	T-M	B-L	T-L	T-L	M-L
#6	Y	3,2	B-M	M-R	M-M	M-M	M-L
#7	Y	2,3	M-R	B-R	M-R	M-R	T-M
#8	Y	3,2	B-R	B-L	B-R	B-R	T-M
#9	Y	3,4	T-R	T-L	M-R	T-M	M-L
#10	Y	4,3	B-L	T-L	B-M	M-L	T-M
#11	N	–,–	M-M	B-M	M-R	B-M	T-L
#12	N	–,–	B-L	M-R	M-M	M-R	T-L
#13	N	–,–	T-R	T-M	B-R	T-M	M-L
#14	N	–,–	T-L	B-M	M-M	B-M	T-R

What the raw evidence shows

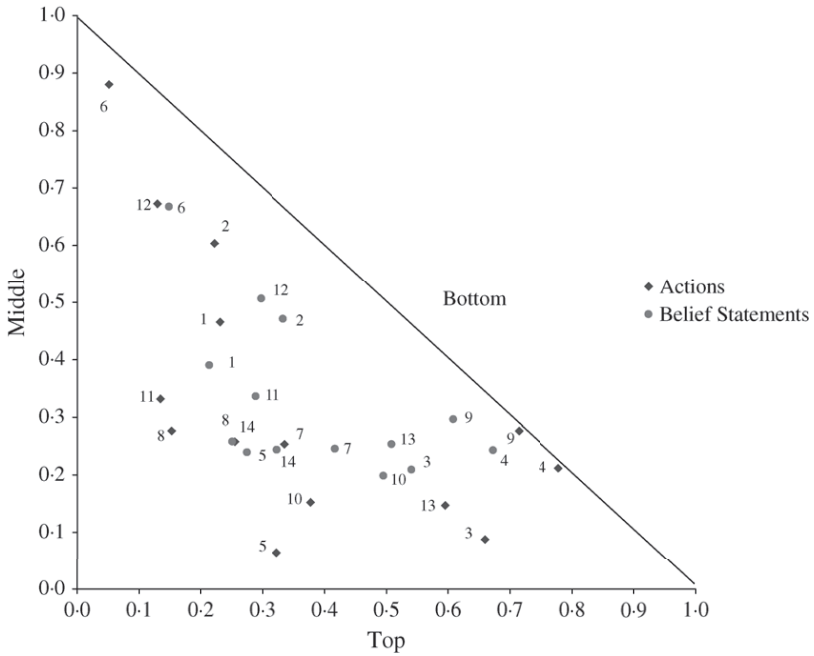


FIGURE 3

Action frequencies and average belief statements (pooled across treatments and isomorphic player roles, numbering according to Row player's perspective)

Left: actions and stated beliefs vary substantially across games.

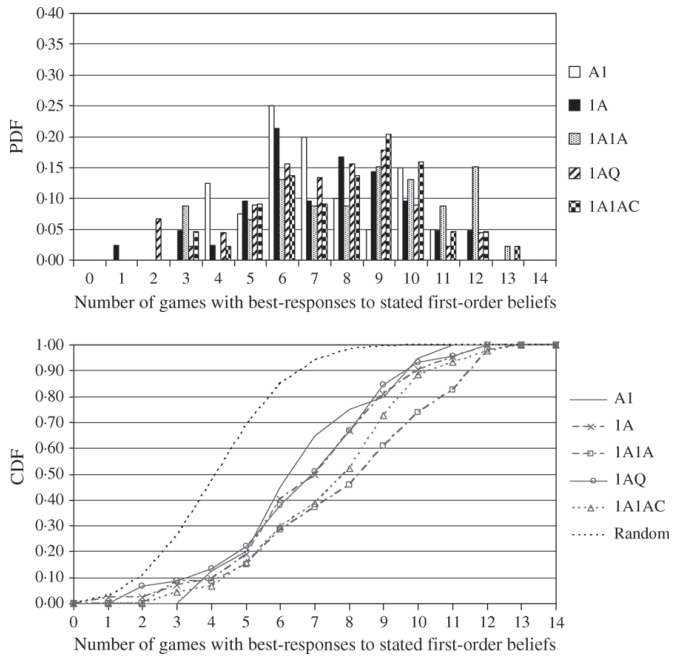


FIGURE 4

(a) Empirical p.d.f. and (b) empirical c.d.f. of number of subjects with x best-responses to stated first-order beliefs

Right: only about half of observed actions are best responses to stated beliefs.

Descriptive result

Subjects' stated beliefs are not a reliable sufficient statistic for their actions: many choices are inconsistent with best response to the beliefs subjects report.

A useful interpretation of the inconsistency

Action choice

A useful interpretation of the inconsistency

Action choice

- Many games in the experiment contain a dominance-based shortcut.

A useful interpretation of the inconsistency

Action choice

- Many games in the experiment contain a dominance-based shortcut.
- As in Section 1, this tends to generate behavior close to D1 / level-1 choice.

A useful interpretation of the inconsistency

Action choice

- Many games in the experiment contain a dominance-based shortcut.
- As in Section 1, this tends to generate behavior close to D1 / level-1 choice.
- Subjects avoid their own dominated option and best respond to a diffuse, weakly sophisticated opponent.

Stated beliefs

A useful interpretation of the inconsistency

Action choice

- Many games in the experiment contain a dominance-based shortcut.
- As in Section 1, this tends to generate behavior close to D1 / level-1 choice.
- Subjects avoid their own dominated option and best respond to a diffuse, weakly sophisticated opponent.

Stated beliefs

- Beliefs are elicited from the subject's own perspective.

A useful interpretation of the inconsistency

Action choice

- Many games in the experiment contain a dominance-based shortcut.
- As in Section 1, this tends to generate behavior close to D1 / level-1 choice.
- Subjects avoid their own dominated option and best respond to a diffuse, weakly sophisticated opponent.

Stated beliefs

- Beliefs are elicited from the subject's own perspective.
- A natural response is: *"What would I do in the opponent's role?"*

A useful interpretation of the inconsistency

Action choice

- Many games in the experiment contain a dominance-based shortcut.
- As in Section 1, this tends to generate behavior close to D1 / level-1 choice.
- Subjects avoid their own dominated option and best respond to a diffuse, weakly sophisticated opponent.

Stated beliefs

- Beliefs are elicited from the subject's own perspective.
- A natural response is: *"What would I do in the opponent's role?"*
- That produces a belief report that is self-anchored and one-step strategic, rather than a literal level-0 uniform prior.

Interpretation for this lecture

Actions can look level-1, and stated beliefs can still fail to justify those actions, because the stated beliefs are not the literal level-0 beliefs required by the textbook level-1 model.

Model comparison: Actions

Abbreviations NE = Nash Eq.; L1 = Level 1; D1 = Level 1 after elimination of dominated strategies; L2 = Level 2; LE = Logit Eq.; ALE = Asymmetric Logit Eq.; NI = Noisy Introspection

TABLE 9

Estimates of low-parameter models using action data

	NE		L1		D1		L2		Opt.		LE		ALE			NI		
	λ^a	ln L	λ^a	ln L	λ^a	ln L	λ^a	ln L	λ^a	ln L	λ^a	ln L	λ^a	$\tilde{\lambda}^a$	ln L	λ^a	$\tilde{\lambda}^a$	ln L
A1	0·60	−611·66	6·13	−541·90	3·73	−571·94	1·31	−593·08	0·65	−607·91	3·34	−565·87	6·13	0·00	−541·90	6·48	1·17	−539·11
1A	0·53	−643·01	7·60	−540·90	3·53	−602·03	1·46	−618·89	0·90	−628·22	3·73	−581·18	7·65	0·00	−540·83	7·65	0·69	−539·75
1A1A	0·75	−699·82	7·09	−602·36	3·84	−652·99	1·49	−676·66	0·84	−694·96	3·87	−637·33	7·10	0·00	−602·35	7·23	1·01	−599·56
1AQ	1·00	−680·15	6·27	−609·69	3·60	−644·64	2·26	−629·39	0·59	−685·05	4·81	−585·98	8·41	0·70	−555·18	8·93	2·21	−542·35
1A1AC	0·57	−672·97	7·85	−556·83	3·48	−633·05	1·34	−652·37	0·59	−669·99	3·34	−625·96	5·94	0·00	−603·99	6·10	1·27	−600·33
Pooled	0·69	−3310·02	6·99	−2855·22	3·63	−3105·03	1·57	−3177·03	0·71	−3288·29	3·88	−3005·67	7·01	0·00	−2853·47	7·19	1·25	−2832·80

ALE, Asymmetric Logit Equilibrium; L1, Level-1; L2 Level-2; LE, Logit Equilibrium; NE, Nash equilibrium, NI, Noisy Introspection; Opt., Optimistic.

Actions

Model comparison: Actions

Abbreviations NE = Nash Eq.; L1 = Level 1; D1 = Level 1 after elimination of dominated strategies; L2 = Level 2; LE = Logit Eq.; ALE = Asymmetric Logit Eq.; NI = Noisy Introspection

TABLE 9

Estimates of low-parameter models using action data

	NE		L1		D1		L2		Opt.		LE		ALE			NI		
	λ^a	ln L	λ^a	ln L	λ^a	ln L	λ^a	ln L	λ^a	ln L	λ^a	ln L	λ^a	$\tilde{\lambda}^a$	ln L	λ^a	$\tilde{\lambda}^a$	ln L
A1	0·60	−611·66	6·13	−541·90	3·73	−571·94	1·31	−593·08	0·65	−607·91	3·34	−565·87	6·13	0·00	−541·90	6·48	1·17	−539·11
1A	0·53	−643·01	7·60	−540·90	3·53	−602·03	1·46	−618·89	0·90	−628·22	3·73	−581·18	7·65	0·00	−540·83	7·65	0·69	−539·75
1A1A	0·75	−699·82	7·09	−602·36	3·84	−652·99	1·49	−676·66	0·84	−694·96	3·87	−637·33	7·10	0·00	−602·35	7·23	1·01	−599·56
1AQ	1·00	−680·15	6·27	−609·69	3·60	−644·64	2·26	−629·39	0·59	−685·05	4·81	−585·98	8·41	0·70	−555·18	8·93	2·21	−542·35
1A1AC	0·57	−672·97	7·85	−556·83	3·48	−633·05	1·34	−652·37	0·59	−669·99	3·34	−625·96	5·94	0·00	−603·99	6·10	1·27	−600·33
Pooled	0·69	−3310·02	6·99	−2855·22	3·63	−3105·03	1·57	−3177·03	0·71	−3288·29	3·88	−3005·67	7·01	0·00	−2853·47	7·19	1·25	−2832·80

ALE, Asymmetric Logit Equilibrium; L1, Level-1; L2 Level-2; LE, Logit Equilibrium; NE, Nash equilibrium, NI, Noisy Introspection; Opt., Optimistic.

Actions

- NI fits best overall

Model comparison: Actions

Abbreviations NE = Nash Eq.; L1 = Level 1; D1 = Level 1 after elimination of dominated strategies; L2 = Level 2; LE = Logit Eq.; ALE = Asymmetric Logit Eq.; NI = Noisy Introspection

TABLE 9

Estimates of low-parameter models using action data

	NE		L1		D1		L2		Opt.		LE		ALE			NI		
	λ^a	ln L	λ^a	ln L	λ^a	ln L	λ^a	ln L	λ^a	ln L	λ^a	ln L	λ^a	$\tilde{\lambda}^a$	ln L	λ^a	$\tilde{\lambda}^a$	ln L
A1	0·60	−611·66	6·13	−541·90	3·73	−571·94	1·31	−593·08	0·65	−607·91	3·34	−565·87	6·13	0·00	−541·90	6·48	1·17	−539·11
1A	0·53	−643·01	7·60	−540·90	3·53	−602·03	1·46	−618·89	0·90	−628·22	3·73	−581·18	7·65	0·00	−540·83	7·65	0·69	−539·75
1A1A	0·75	−699·82	7·09	−602·36	3·84	−652·99	1·49	−676·66	0·84	−694·96	3·87	−637·33	7·10	0·00	−602·35	7·23	1·01	−599·56
1AQ	1·00	−680·15	6·27	−609·69	3·60	−644·64	2·26	−629·39	0·59	−685·05	4·81	−585·98	8·41	0·70	−555·18	8·93	2·21	−542·35
1A1AC	0·57	−672·97	7·85	−556·83	3·48	−633·05	1·34	−652·37	0·59	−669·99	3·34	−625·96	5·94	0·00	−603·99	6·10	1·27	−600·33
Pooled	0·69	−3310·02	6·99	−2855·22	3·63	−3105·03	1·57	−3177·03	0·71	−3288·29	3·88	−3005·67	7·01	0·00	−2853·47	7·19	1·25	−2832·80

ALE, Asymmetric Logit Equilibrium; L1, Level-1; L2 Level-2; LE, Logit Equilibrium; NE, Nash equilibrium, NI, Noisy Introspection; Opt., Optimistic.

Actions

- NI fits best overall
- ALE and L1 are close contenders

Model comparison: Actions

Abbreviations NE = Nash Eq.; L1 = Level 1; D1 = Level 1 after elimination of dominated strategies; L2 = Level 2; LE = Logit Eq.; ALE = Asymmetric Logit Eq.; NI = Noisy Introspection

TABLE 9

Estimates of low-parameter models using action data

	NE		L1		D1		L2		Opt.		LE		ALE			NI		
	λ^a	ln L	λ^a	ln L	λ^a	ln L	λ^a	ln L	λ^a	ln L	λ^a	ln L	λ^a	$\tilde{\lambda}^a$	ln L	λ^a	$\tilde{\lambda}^a$	ln L
A1	0·60	−611·66	6·13	−541·90	3·73	−571·94	1·31	−593·08	0·65	−607·91	3·34	−565·87	6·13	0·00	−541·90	6·48	1·17	−539·11
1A	0·53	−643·01	7·60	−540·90	3·53	−602·03	1·46	−618·89	0·90	−628·22	3·73	−581·18	7·65	0·00	−540·83	7·65	0·69	−539·75
1A1A	0·75	−699·82	7·09	−602·36	3·84	−652·99	1·49	−676·66	0·84	−694·96	3·87	−637·33	7·10	0·00	−602·35	7·23	1·01	−599·56
1AQ	1·00	−680·15	6·27	−609·69	3·60	−644·64	2·26	−629·39	0·59	−685·05	4·81	−585·98	8·41	0·70	−555·18	8·93	2·21	−542·35
1A1AC	0·57	−672·97	7·85	−556·83	3·48	−633·05	1·34	−652·37	0·59	−669·99	3·34	−625·96	5·94	0·00	−603·99	6·10	1·27	−600·33
Pooled	0·69	−3310·02	6·99	−2855·22	3·63	−3105·03	1·57	−3177·03	0·71	−3288·29	3·88	−3005·67	7·01	0·00	−2853·47	7·19	1·25	−2832·80

ALE, Asymmetric Logit Equilibrium; L1, Level-1; L2 Level-2; LE, Logit Equilibrium; NE, Nash equilibrium, NI, Noisy Introspection; Opt., Optimistic.

Actions

- NI fits best overall
- ALE and L1 are close contenders
- the remaining models fit clearly worse

Model comparison: Beliefs

Abbreviations NE = Nash Eq.; L1 = Level 1; D1 = Level 1 after elimination of dominated strategies; L2 = Level 2; LE = Logit Eq.; ALE = Asymmetric Logit Eq.; NI = Noisy Introspection

TABLE 10

Estimates of low-parameter models using belief statement data

	NE		L1		D1		L2		Opt.		LE			ALE				NI			
	λ^{bs}	ln L	λ^{bs}	ln L	λ^{bs}	ln L	λ^{bs}	ln L	λ^{bs}	ln L	λ^{a}	λ^{bs}	ln L	λ^{a}	$\tilde{\lambda}^{a}$	λ^{bs}	ln L	λ^{a}	$\tilde{\lambda}^{a}$	λ^{bs}	ln L
A1	0·00	−4769·63	0·00	−4769·63	0·00	−4769·63	0·18	−4717·64	0·00	−4769·63	7·59	0·00	−4769·63	0·00	28·20	0·20	−4716·89	87·19	20·06	0·20	−4748·54
1A	0·00	−5008·11	0·00	−5008·11	0·04	−5007·74	0·23	−4924·41	0·00	−5008·11	8·08	0·11	−5000·79	0·00	21·70	0·28	−4921·45	76·76	18·42	0·35	−4943·00
1A1A	0·00	−5485·07	0·00	−5485·07	0·00	−5485·07	0·22	−5402·75	0·00	−5485·07	9·02	0·00	−5483·38	0·03	40·10	0·22	−5402·56	96·65	24·17	0·26	−5443·67
1AQ	0·00	−5365·83	0·00	−5365·83	0·00	−5365·83	0·29	−5233·33	0·00	−5365·83	8·09	0·18	−5348·37	0·07	27·22	0·32	−5230·81	75·33	18·65	0·41	−5273·67
1A1AC	0·00	−5246·59	0·00	−5246·59	0·10	−5243·82	0·20	−5177·45	0·00	−5246·59	8·29	0·06	−5244·40	0·00	28·76	0.22	−5176·13	62·49	15·47	0·26	−5205·22
Pooled	0·00	−25875·2	0·00	−25875·2	0·00	−25875·2	0·22	−25464·6	0·00	−25875·2	8·09	0·07	−25860·9	0·00	28·76	0·24	−25458·6	79·79	19·64	0·29	−25630·0

ALE, Asymmetric Logit Equilibrium; L1, Level-1; L2, Level-2; LE, Logit Equilibrium; NE, Nash equilibrium, NI, Noisy Introspection; Opt., Optimistic.

Beliefs

Model comparison: Beliefs

Abbreviations NE = Nash Eq.; L1 = Level 1; D1 = Level 1 after elimination of dominated strategies; L2 = Level 2; LE = Logit Eq.; ALE = Asymmetric Logit Eq.; NI = Noisy Introspection

TABLE 10

Estimates of low-parameter models using belief statement data

	NE		L1		D1		L2		Opt.		LE			ALE				NI			
	λ^{bs}	ln L	λ^{bs}	ln L	λ^{bs}	ln L	λ^{bs}	ln L	λ^{bs}	ln L	λ^{a}	λ^{bs}	ln L	λ^{a}	$\tilde{\lambda}^{a}$	λ^{bs}	ln L	λ^{a}	$\tilde{\lambda}^{a}$	λ^{bs}	ln L
A1	0·00	−4769·63	0·00	−4769·63	0·00	−4769·63	0·18	−4717·64	0·00	−4769·63	7·59	0·00	−4769·63	0·00	28·20	0·20	−4716·89	87·19	20·06	0·20	−4748·54
1A	0·00	−5008·11	0·00	−5008·11	0·04	−5007·74	0·23	−4924·41	0·00	−5008·11	8·08	0·11	−5000·79	0·00	21·70	0·28	−4921·45	76·76	18·42	0·35	−4943·00
1A1A	0·00	−5485·07	0·00	−5485·07	0·00	−5485·07	0·22	−5402·75	0·00	−5485·07	9·02	0·00	−5483·38	0·03	40·10	0·22	−5402·56	96·65	24·17	0·26	−5443·67
1AQ	0·00	−5365·83	0·00	−5365·83	0·00	−5365·83	0·29	−5233·33	0·00	−5365·83	8·09	0·18	−5348·37	0·07	27·22	0·32	−5230·81	75·33	18·65	0·41	−5273·67
1A1AC	0·00	−5246·59	0·00	−5246·59	0·10	−5243·82	0·20	−5177·45	0·00	−5246·59	8·29	0·06	−5244·40	0·00	28·76	0.22	−5176·13	62·49	15·47	0·26	−5205·22
Pooled	0·00	−25875·2	0·00	−25875·2	0·00	−25875·2	0·22	−25464·6	0·00	−25875·2	8·09	0·07	−25860·9	0·00	28·76	0·24	−25458·6	79·79	19·64	0·29	−25630·0

ALE, Asymmetric Logit Equilibrium; L1, Level-1; L2, Level-2; LE, Logit Equilibrium; NE, Nash equilibrium, NI, Noisy Introspection; Opt., Optimistic.

Beliefs

- ALE and L2 fit best

Model comparison: Beliefs

Abbreviations NE = Nash Eq.; L1 = Level 1; D1 = Level 1 after elimination of dominated strategies; L2 = Level 2; LE = Logit Eq.; ALE = Asymmetric Logit Eq.; NI = Noisy Introspection

TABLE 10

Estimates of low-parameter models using belief statement data

	NE		L1		D1		L2		Opt.		LE			ALE				NI			
	λ^{bs}	ln L	λ^{bs}	ln L	λ^{bs}	ln L	λ^{bs}	ln L	λ^{bs}	ln L	λ^{a}	λ^{bs}	ln L	λ^{a}	$\tilde{\lambda}^{a}$	λ^{bs}	ln L	λ^{a}	$\tilde{\lambda}^{a}$	λ^{bs}	ln L
A1	0·00	−4769·63	0·00	−4769·63	0·00	−4769·63	0·18	−4717·64	0·00	−4769·63	7·59	0·00	−4769·63	0·00	28·20	0·20	−4716·89	87·19	20·06	0·20	−4748·54
1A	0·00	−5008·11	0·00	−5008·11	0·04	−5007·74	0·23	−4924·41	0·00	−5008·11	8·08	0·11	−5000·79	0·00	21·70	0·28	−4921·45	76·76	18·42	0·35	−4943·00
1A1A	0·00	−5485·07	0·00	−5485·07	0·00	−5485·07	0·22	−5402·75	0·00	−5485·07	9·02	0·00	−5483·38	0·03	40·10	0·22	−5402·56	96·65	24·17	0·26	−5443·67
1AQ	0·00	−5365·83	0·00	−5365·83	0·00	−5365·83	0·29	−5233·33	0·00	−5365·83	8·09	0·18	−5348·37	0·07	27·22	0·32	−5230·81	75·33	18·65	0·41	−5273·67
1A1AC	0·00	−5246·59	0·00	−5246·59	0·10	−5243·82	0·20	−5177·45	0·00	−5246·59	8·29	0·06	−5244·40	0·00	28·76	0.22	−5176·13	62·49	15·47	0·26	−5205·22
Pooled	0·00	−25875·2	0·00	−25875·2	0·00	−25875·2	0·22	−25464·6	0·00	−25875·2	8·09	0·07	−25860·9	0·00	28·76	0·24	−25458·6	79·79	19·64	0·29	−25630·0

ALE, Asymmetric Logit Equilibrium; L1, Level-1; L2, Level-2; LE, Logit Equilibrium; NE, Nash equilibrium, NI, Noisy Introspection; Opt., Optimistic.

Beliefs

- ALE and L2 fit best
- NI is still reasonable

Model comparison: Beliefs

Abbreviations NE = Nash Eq.; L1 = Level 1; D1 = Level 1 after elimination of dominated strategies; L2 = Level 2; LE = Logit Eq.; ALE = Asymmetric Logit Eq.; NI = Noisy Introspection

TABLE 10

Estimates of low-parameter models using belief statement data

	NE		L1		D1		L2		Opt.		LE			ALE				NI			
	λ^{bs}	ln L	λ^{bs}	ln L	λ^{bs}	ln L	λ^{bs}	ln L	λ^{bs}	ln L	λ^{a}	λ^{bs}	ln L	λ^{a}	$\tilde{\lambda}^{a}$	λ^{bs}	ln L	λ^{a}	$\tilde{\lambda}^{a}$	λ^{bs}	ln L
A1	0·00	−4769·63	0·00	−4769·63	0·00	−4769·63	0·18	−4717·64	0·00	−4769·63	7·59	0·00	−4769·63	0·00	28·20	0·20	−4716·89	87·19	20·06	0·20	−4748·54
1A	0·00	−5008·11	0·00	−5008·11	0·04	−5007·74	0·23	−4924·41	0·00	−5008·11	8·08	0·11	−5000·79	0·00	21·70	0·28	−4921·45	76·76	18·42	0·35	−4943·00
1A1A	0·00	−5485·07	0·00	−5485·07	0·00	−5485·07	0·22	−5402·75	0·00	−5485·07	9·02	0·00	−5483·38	0·03	40·10	0·22	−5402·56	96·65	24·17	0·26	−5443·67
1AQ	0·00	−5365·83	0·00	−5365·83	0·00	−5365·83	0·29	−5233·33	0·00	−5365·83	8·09	0·18	−5348·37	0·07	27·22	0·32	−5230·81	75·33	18·65	0·41	−5273·67
1A1AC	0·00	−5246·59	0·00	−5246·59	0·10	−5243·82	0·20	−5177·45	0·00	−5246·59	8·29	0·06	−5244·40	0·00	28·76	0.22	−5176·13	62·49	15·47	0·26	−5205·22
Pooled	0·00	−25875·2	0·00	−25875·2	0·00	−25875·2	0·22	−25464·6	0·00	−25875·2	8·09	0·07	−25860·9	0·00	28·76	0·24	−25458·6	79·79	19·64	0·29	−25630·0

ALE, Asymmetric Logit Equilibrium; L1, Level-1; L2, Level-2; LE, Logit Equilibrium; NE, Nash equilibrium, NI, Noisy Introspection; Opt., Optimistic.

Beliefs

- ALE and L2 fit best
- NI is still reasonable
- literal low-level belief benchmarks fit poorly

What to take from the horse race

Relative to actions, stated beliefs look more sophisticated. This is exactly why best-response consistency between actions and stated beliefs is low.

Do level-1 players believe in uniform randomization?

Do level-1 players believe in uniform randomization?

- No, not literally. A good fit of level-1 actions does not imply literal level-0 uniform beliefs.

Do level-1 players believe in uniform randomization?

- No, not literally. A good fit of level-1 actions does not imply literal level-0 uniform beliefs.
- In these data, actions are often close to D1 / level-1 because many games allow a simple dominance-based shortcut.

Do level-1 players believe in uniform randomization?

- No, not literally. A good fit of level-1 actions does not imply literal level-0 uniform beliefs.
- In these data, actions are often close to D1 / level-1 because many games allow a simple dominance-based shortcut.
- Stated beliefs appear to be formed from a more self-anchored perspective: "What would I do in the opponent's position?"

Do level-1 players believe in uniform randomization?

- No, not literally. A good fit of level-1 actions does not imply literal level-0 uniform beliefs.
- In these data, actions are often close to D1 / level-1 because many games allow a simple dominance-based shortcut.
- Stated beliefs appear to be formed from a more self-anchored perspective: "What would I do in the opponent's position?"
- Therefore belief-action inconsistency is not a side issue; it is evidence that the belief object behind level-1 should not be read too literally.

Transition

We now have both pieces of the answer: *when* choice looks level-1 and *why* the underlying beliefs need not be literal level-0 beliefs. The final step is to put the lecture together.

Conclusion

General patterns in strategic behavior

What have we learned?

This lecture asked two organizing questions.

1. When level-k, when QRE?

What have we learned?

This lecture asked two organizing questions.

1. When level-k, when QRE?
   - If a game contains dominated actions, observed choice is often essentially level-1 / D1 behavior.

What have we learned?

This lecture asked two organizing questions.

1. When level-k, when QRE?
   - If a game contains dominated actions, observed choice is often essentially level-1 / D1 behavior.
   - If a game contains no dominated actions, observed choice is often closer to logit equilibrium / QRE-type response.
2. Do level-1 players really believe opponents randomize uniformly?

What have we learned?

This lecture asked two organizing questions.

1. When level-k, when QRE?
   - If a game contains dominated actions, observed choice is often essentially level-1 / D1 behavior.
   - If a game contains no dominated actions, observed choice is often closer to logit equilibrium / QRE-type response.
2. Do level-1 players really believe opponents randomize uniformly?
   - Not literally.

What have we learned?

This lecture asked two organizing questions.

1. When level-k, when QRE?
   - If a game contains dominated actions, observed choice is often essentially level-1 / D1 behavior.
   - If a game contains no dominated actions, observed choice is often closer to logit equilibrium / QRE-type response.
2. Do level-1 players really believe opponents randomize uniformly?
   - Not literally.
   - A good fit of level-1 actions does not imply literal level-0 beliefs.

What have we learned?

This lecture asked two organizing questions.

1. When level-k, when QRE?
   - If a game contains dominated actions, observed choice is often essentially level-1 / D1 behavior.
   - If a game contains no dominated actions, observed choice is often closer to logit equilibrium / QRE-type response.
2. Do level-1 players really believe opponents randomize uniformly?
   - Not literally.
   - A good fit of level-1 actions does not imply literal level-0 beliefs.
   - In the belief-elicitation data, stated beliefs appear more self-anchored and more sophisticated than the actions they are supposed to justify.

Why ALE is useful

ALE is not just another behavioral model. It gives a single framework that connects the main empirical benchmarks.

Level-1 $\longleftrightarrow$ ALE $\longleftrightarrow$ Logit Eq./QRE $\longleftrightarrow$ Nash

Why ALE is useful

ALE is not just another behavioral model. It gives a single framework that connects the main empirical benchmarks.

Level-1 $\longleftrightarrow$ ALE $\longleftrightarrow$ Logit Eq./QRE $\longleftrightarrow$ Nash

- ALE allows own payoff sensitivity and perceived opponent sensitivity to differ.

Why ALE is useful

ALE is not just another behavioral model. It gives a single framework that connects the main empirical benchmarks.

Level-1 $\longleftrightarrow$ ALE $\longleftrightarrow$ Logit Eq./QRE $\longleftrightarrow$ Nash

- ALE allows own payoff sensitivity and perceived opponent sensitivity to differ.
- It therefore lets us interpret behavior without imposing symmetric rational expectations.

Why ALE is useful

ALE is not just another behavioral model. It gives a single framework that connects the main empirical benchmarks.

Level-1 $\longleftrightarrow$ ALE $\longleftrightarrow$ Logit Eq./QRE $\longleftrightarrow$ Nash

- ALE allows own payoff sensitivity and perceived opponent sensitivity to differ.
- It therefore lets us interpret behavior without imposing symmetric rational expectations.
- It turns "when level-1, when QRE?" into an empirical question, rather than a modeling choice made in advance.

This broader perspective on nonequilibrium strategic thinking is also emphasized by Crawford et al. (2013).

What this means for the next lecture

Today's message was deliberately structural, not just descriptive.

What this means for the next lecture

Today's message was deliberately structural, not just descriptive.

- Strategic behavior is often neither literal Nash play nor well described by one universal finite level-k model.

What this means for the next lecture

Today's message was deliberately structural, not just descriptive.

- Strategic behavior is often neither literal Nash play nor well described by one universal finite level-k model.
- Instead, observed choice varies systematically with game structure, especially with the presence or absence of dominated actions.

What this means for the next lecture

Today's message was deliberately structural, not just descriptive.

- Strategic behavior is often neither literal Nash play nor well described by one universal finite level-k model.
- Instead, observed choice varies systematically with game structure, especially with the presence or absence of dominated actions.
- This suggests that we need models allowing for heterogeneity in reasoning and for different forms of strategic response across environments.

Bridge to Lecture 06

Lecture 06 asks how to model this variation more flexibly: heterogeneous strategic types, richer fixed-point models of noisy response, and alternative ways to organize aggregate play in normal-form games.

References

General patterns in strategic behavior

References and further reading

Core papers for this lecture

References and further reading

Core papers for this lecture

- Weizsäcker, 2003: asymmetric logit equilibrium and the role of dominated actions

References and further reading

Core papers for this lecture

- Weizsäcker, 2003: asymmetric logit equilibrium and the role of dominated actions
- Costa-Gomes and Weizsäcker, 2008: stated beliefs, actions, and belief-action inconsistency

Lecture-wide perspective

References and further reading

Core papers for this lecture

- Weizsäcker, 2003: asymmetric logit equilibrium and the role of dominated actions
- Costa-Gomes and Weizsäcker, 2008: stated beliefs, actions, and belief-action inconsistency

Lecture-wide perspective

- Crawford et al., 2013: structural models of nonequilibrium strategic thinking

References and further reading

Core papers for this lecture

- Weizsäcker, 2003: asymmetric logit equilibrium and the role of dominated actions
- Costa-Gomes and Weizsäcker, 2008: stated beliefs, actions, and belief-action inconsistency

Lecture-wide perspective

- Crawford et al., 2013: structural models of nonequilibrium strategic thinking
- Breitmoser, 2012: bridge to the beauty-contest evidence earlier in the course

Useful follow-up reading

References and further reading

Core papers for this lecture

- Weizsäcker, 2003: asymmetric logit equilibrium and the role of dominated actions
- Costa-Gomes and Weizsäcker, 2008: stated beliefs, actions, and belief-action inconsistency

Lecture-wide perspective

- Crawford et al., 2013: structural models of nonequilibrium strategic thinking
- Breitmoser, 2012: bridge to the beauty-contest evidence earlier in the course

Useful follow-up reading

- Hyndman et al., 2022: stability of beliefs and actions within and across games

References and further reading

Core papers for this lecture

- Weizsäcker, 2003: asymmetric logit equilibrium and the role of dominated actions
- Costa-Gomes and Weizsäcker, 2008: stated beliefs, actions, and belief-action inconsistency

Lecture-wide perspective

- Crawford et al., 2013: structural models of nonequilibrium strategic thinking
- Breitmoser, 2012: bridge to the beauty-contest evidence earlier in the course

Useful follow-up reading

- Hyndman et al., 2022: stability of beliefs and actions within and across games
- Wolff and Folli, 2024: belief uncertainty as a source of low consistency

How to use these papers

References and further reading

Core papers for this lecture

- Weizsäcker, 2003: asymmetric logit equilibrium and the role of dominated actions
- Costa-Gomes and Weizsäcker, 2008: stated beliefs, actions, and belief-action inconsistency

Lecture-wide perspective

- Crawford et al., 2013: structural models of nonequilibrium strategic thinking
- Breitmoser, 2012: bridge to the beauty-contest evidence earlier in the course

Useful follow-up reading

- Hyndman et al., 2022: stability of beliefs and actions within and across games
- Wolff and Folli, 2024: belief uncertainty as a source of low consistency

How to use these papers

- The lecture is self-contained: the papers are not required to follow the main line.

References and further reading

Core papers for this lecture

- Weizsäcker, 2003: asymmetric logit equilibrium and the role of dominated actions
- Costa-Gomes and Weizsäcker, 2008: stated beliefs, actions, and belief-action inconsistency

Lecture-wide perspective

- Crawford et al., 2013: structural models of nonequilibrium strategic thinking
- Breitmoser, 2012: bridge to the beauty-contest evidence earlier in the course

Useful follow-up reading

- Hyndman et al., 2022: stability of beliefs and actions within and across games
- Wolff and Folli, 2024: belief uncertainty as a source of low consistency

How to use these papers

- The lecture is self-contained: the papers are not required to follow the main line.

References I

Breitmoser, Y. (2012). Strategic reasoning in p-beauty contests. *Games and Economic Behavior*, 75(2):555–569.

Costa-Gomes, M. A., Crawford, V. P., and Broseta, B. (2001). Cognition and behavior in normal-form games: An experimental study. *Econometrica*, 69(5):1193–1235.

Costa-Gomes, M. A. and Weizsäcker, G. (2008). Stated beliefs and play in normal-form games. *The Review of Economic Studies*, 75(3):729–762.

Crawford, V. P., Costa-Gomes, M. A., and Iriberri, N. (2013). Structural models of nonequilibrium strategic thinking: Theory, evidence, and applications. *Journal of Economic Literature*, 51(1):5–62.

Hyndman, K., Terracol, A., and Vaksmann, J. (2022). Beliefs and (in)stability in normal-form games. *Experimental Economics*, 25(4):1146–1172.

Stahl, D. O. and Wilson, P. W. (1994). Experimental evidence on players' models of other players. *Journal of Economic Behavior & Organization*, 25(3):309–327.

Stahl, D. O. and Wilson, P. W. (1995). On players' models of other players: Theory and experimental evidence. *Games and Economic Behavior*, 10(1):218–254.

Weizsäcker, G. (2003). Ignoring the rationality of others: Evidence from experimental normal-form games. *Games and Economic Behavior*, 44(1):145–171.

Wolff, I. and Folli, D. (2024). Why is belief–action consistency so low? the role of belief uncertainty. *Journal of Economic Behavior & Organization*, 227:106722.

Repetition

General patterns in strategic behavior

Repetition I: core concepts

Repetition I: core concepts

1. **What is the lecture's main classification question?**
   Answer sketch: When do data look essentially like level-1 behavior, and when do they look more like logit equilibrium / QRE?

Repetition I: core concepts

1. **What is the lecture's main classification question?**
   Answer sketch: When do data look essentially like level-1 behavior, and when do they look more like logit equilibrium / QRE?

2. **What is asymmetric logit equilibrium (ALE)?**
   Answer sketch: A player logit-responds to a perceived opponent logit response, allowing own precision and perceived opponent precision to differ.

Repetition I: core concepts

1. **What is the lecture's main classification question?**
   Answer sketch: When do data look essentially like level-1 behavior, and when do they look more like logit equilibrium / QRE?
2. **What is asymmetric logit equilibrium (ALE)?**
   Answer sketch: A player logit-responds to a perceived opponent logit response, allowing own precision and perceived opponent precision to differ.
3. **How does ALE relate to level-1, QRE, and Nash equilibrium?**
   Answer sketch: ALE is a continuum containing level-1, QRE, and Nash as special or limiting cases.

Repetition I: core concepts

1. **What is the lecture's main classification question?**
   Answer sketch: When do data look essentially like level-1 behavior, and when do they look more like logit equilibrium / QRE?

2. **What is asymmetric logit equilibrium (ALE)?**
   Answer sketch: A player logit-responds to a perceived opponent logit response, allowing own precision and perceived opponent precision to differ.

3. **How does ALE relate to level-1, QRE, and Nash equilibrium?**
   Answer sketch: ALE is a continuum containing level-1, QRE, and Nash as special or limiting cases.

4. **What does $\lambda_i > \tilde{\lambda}_j$ mean behaviorally?**
   Answer sketch: I respond more sharply to my own incentives than I think you respond to yours.

Repetition II: evidence from actions

Repetition II: evidence from actions

5. **What is the baseline result in Weizsäcker (2003)?**
   Answer sketch: Subjects behave as if they underestimate opponents' rationality, so symmetric QRE is too restrictive.

Repetition II: evidence from actions

5. **What is the baseline result in Weizsäcker (2003)?**
   Answer sketch: Subjects behave as if they underestimate opponents' rationality, so symmetric QRE is too restrictive.

6. **Why are dominated actions the key divider?**
   Answer sketch: Dominated actions create a simple shortcut: players often eliminate their own dominated option without fully attributing the same reasoning to opponents.

Repetition II: evidence from actions

5. **What is the baseline result in Weizsäcker (2003)?**
   Answer sketch: Subjects behave as if they underestimate opponents' rationality, so symmetric QRE is too restrictive.

6. **Why are dominated actions the key divider?**
   Answer sketch: Dominated actions create a simple shortcut: players often eliminate their own dominated option without fully attributing the same reasoning to opponents.

7. **What is the lecture's main empirical rule?**
   Answer sketch: Dominated actions $\Rightarrow$ essentially level-1 / D1 behavior; no dominated actions $\Rightarrow$ essentially logit-equilibrium behavior.

Repetition II: evidence from actions

5. **What is the baseline result in Weizsäcker (2003)?**
   Answer sketch: Subjects behave as if they underestimate opponents' rationality, so symmetric QRE is too restrictive.

6. **Why are dominated actions the key divider?**
   Answer sketch: Dominated actions create a simple shortcut: players often eliminate their own dominated option without fully attributing the same reasoning to opponents.

7. **What is the lecture's main empirical rule?**
   Answer sketch: Dominated actions $\Rightarrow$ essentially level-1 / D1 behavior; no dominated actions $\Rightarrow$ essentially logit-equilibrium behavior.

8. **Why compare ALE to ANNE?**
   Answer sketch: To distinguish payoff-sensitive noisy response from models that treat deviations mainly as random trembles.

Repetition III: beliefs and interpretation

Repetition III: beliefs and interpretation

9. **What does belief elicitation add beyond action data?**
   Answer sketch: It lets us test directly whether observed actions are actually best responses to what subjects say they believe.

Repetition III: beliefs and interpretation

9. **What does belief elicitation add beyond action data?**
   Answer sketch: It lets us test directly whether observed actions are actually best responses to what subjects say they believe.

10. **What is the main descriptive result of Costa-Gomes and Weizsäcker (2008)?**
    Answer sketch: Subjects fail to best respond to their own stated beliefs in almost half of the games.

Repetition III: beliefs and interpretation

9. **What does belief elicitation add beyond action data?**
   Answer sketch: It lets us test directly whether observed actions are actually best responses to what subjects say they believe.

10. **What is the main descriptive result of Costa-Gomes and Weizsäcker (2008)?**
    Answer sketch: Subjects fail to best respond to their own stated beliefs in almost half of the games.

11. **Why is belief-action inconsistency not just a paradox?**
    Answer sketch: Actions can still look level-1 in dominance-rich games, while stated beliefs may be formed from a self-anchored perspective rather than as literal level-0 beliefs.

Repetition III: beliefs and interpretation

9. **What does belief elicitation add beyond action data?**
   Answer sketch: It lets us test directly whether observed actions are actually best responses to what subjects say they believe.

10. **What is the main descriptive result of Costa-Gomes and Weizsäcker (2008)?**
    Answer sketch: Subjects fail to best respond to their own stated beliefs in almost half of the games.

11. **Why is belief-action inconsistency not just a paradox?**
    Answer sketch: Actions can still look level-1 in dominance-rich games, while stated beliefs may be formed from a self-anchored perspective rather than as literal level-0 beliefs.

12. **Do level-1 players literally believe opponents randomize uniformly?**
    Answer sketch: Not necessarily; in this lecture, level-1 is mainly a useful reduced-form description of choice, not a literal belief model.

Repetition III: beliefs and interpretation

9. **What does belief elicitation add beyond action data?**
   Answer sketch: It lets us test directly whether observed actions are actually best responses to what subjects say they believe.

10. **What is the main descriptive result of Costa-Gomes and Weizsäcker (2008)?**
    Answer sketch: Subjects fail to best respond to their own stated beliefs in almost half of the games.

11. **Why is belief-action inconsistency not just a paradox?**
    Answer sketch: Actions can still look level-1 in dominance-rich games, while stated beliefs may be formed from a self-anchored perspective rather than as literal level-0 beliefs.

12. **Do level-1 players literally believe opponents randomize uniformly?**
    Answer sketch: Not necessarily; in this lecture, level-1 is mainly a useful reduced-form description of choice, not a literal belief model.

13. **How does Lecture 05 connect to the beauty-contest lectures and to Lecture 06?**
    Answer sketch: Like the beauty contest, it shows a tension between one-step reasoning and smooth equilibrium response; the next lecture asks how to model that variation more flexibly.

Asymmetric logit equilibrium (ALE)

Let $r_i(\pi_j)$ denote the logit response with precision λ_i to strategy π_j. Let $\tilde{r}_j(\pi_i)$ denote i's perceived response of j (with perceived precision $\tilde{\lambda}_j$) to π_i. Strategy π_i is an ALE strategy if $\pi_i = r_i(\tilde{r}_j(\pi_i))$.

Table 7
Separate estimations of the ALE strategy model for the data from games with and without dominated strategies, DD and NDD, respectively

Data set	DD	NDD
ρ	0.09	38.70
K	1.27	171.03
$\tilde{\rho}$	1.43	8.04
$\widetilde{K}$	3.03	35.94
l^*	-583.582	-326.213
$E[\lambda^i]$	13.14	4.42
$E[\tilde{\lambda}^j]$	2.12	4.47
sig$(E[\lambda^i] = E[\tilde{\lambda}^j])$	5.045×10^{-9}	0.942
sig$(\rho = \tilde{\rho}, K = \widetilde{K})$	4.647×10^{-12}	0.976
sig$((\bar{\lambda}^i, \bar{\tilde{\lambda}}^j)$ fix over i)	2.924×10^{-29}	0.976
sig$(\bar{\lambda}^i = \bar{\tilde{\lambda}}^j = \lambda$, fix over i)	2.073×10^{-36}	0.997
# of model rejections at $p = 5\%$	7 (out of 28)	4 (out of 9)
# of cases with $\lambda^i > \tilde{\lambda}^j$	106 (29 sig.)	79 (9 sig.)
# of cases with $\lambda^i \leqslant \tilde{\lambda}^j$	54 (0 sig.)	81 (0 sig.)

Note. See Table 3.

Asymmetric logit equilibrium (ALE)

Let $r_i(\pi_j)$ denote the logit response with precision λ_i to strategy π_j. Let $\tilde{r}_j(\pi_i)$ denote i's perceived response of j (with perceived precision $\tilde{\lambda}_j$) to π_i. Strategy π_i is an ALE strategy if $\pi_i = r_i(\tilde{r}_j(\pi_i))$.

Table 7
Separate estimations of the ALE strategy model for the data from games with and without dominated strategies, DD and NDD, respectively

Data set	DD	NDD
ρ	0.09	38.70
K	1.27	171.03
$\tilde{\rho}$	1.43	8.04
$\widetilde{K}$	3.03	35.94
l^*	-583.582	-326.213
$E[\lambda^i]$	13.14	4.42
$E[\tilde{\lambda}^j]$	2.12	4.47
sig$(E[\lambda^i] = E[\tilde{\lambda}^j])$	5.045×10^{-9}	0.942
sig$(\rho = \tilde{\rho}, K = \widetilde{K})$	4.647×10^{-12}	0.976
sig$((\bar{\lambda}^i, \bar{\tilde{\lambda}}^j)$ fix over i)	2.924×10^{-29}	0.976
sig$(\bar{\lambda}^i = \bar{\tilde{\lambda}}^j = \lambda$, fix over i)	2.073×10^{-36}	0.997
# of model rejections at $p = 5\%$	7 (out of 28)	4 (out of 9)
# of cases with $\lambda^i > \tilde{\lambda}^j$	106 (29 sig.)	79 (9 sig.)
# of cases with $\lambda^i \leqslant \tilde{\lambda}^j$	54 (0 sig.)	81 (0 sig.)

Note. See Table 3.

Asymmetric logit equilibrium (ALE)

Let $r_i(\pi_j)$ denote the logit response with precision λ_i to strategy π_j. Let $\tilde{r}_j(\pi_i)$ denote i's perceived response of j (with perceived precision $\tilde{\lambda}_j$) to π_i. Strategy π_i is an ALE strategy if $\pi_i = r_i(\tilde{r}_j(\pi_i))$.

Table 7
Separate estimations of the ALE strategy model for the data from games with and without dominated strategies, DD and NDD, respectively

Data set	DD	NDD
ρ	0.09	38.70
K	1.27	171.03
$\tilde{\rho}$	1.43	8.04
$\widetilde{K}$	3.03	35.94
l^*	-583.582	-326.213
$E[\lambda^i]$	13.14	4.42
$E[\tilde{\lambda}^j]$	2.12	4.47
sig$(E[\lambda^i] = E[\tilde{\lambda}^j])$	5.045×10^{-9}	0.942
sig$(\rho = \tilde{\rho}, K = \widetilde{K})$	4.647×10^{-12}	0.976
sig$((\bar{\lambda}^i, \bar{\tilde{\lambda}}^j)$ fix over i)	2.924×10^{-29}	0.976
sig$(\bar{\lambda}^i = \bar{\tilde{\lambda}}^j = \lambda$, fix over i)	2.073×10^{-36}	0.997
# of model rejections at $p = 5\%$	7 (out of 28)	4 (out of 9)
# of cases with $\lambda^i > \tilde{\lambda}^j$	106 (29 sig.)	79 (9 sig.)
# of cases with $\lambda^i \leqslant \tilde{\lambda}^j$	54 (0 sig.)	81 (0 sig.)

Note. See Table 3.