Games of incomplete information

Summer 26

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Discussion

Topics

- Information cascades and limited look-ahead (Kübler and Weizsäcker, 2004; Goeree et al., 2007; Weizsäcker, 2010)
- Signaling: Brandts and Holt (1992); Kübler et al. (2008); de Haan et al. (2011); Blume et al. (2020)

Information cascades

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Informational cascades

"An informational cascade occurs when it is optimal for an individual, having observed the actions of those ahead of him, to follow the behavior of the preceding individual without regard to his own information."

Bikhchandani, Hirschleifer, and Welch, 1992

Information cascades capture the idea that a group of rational decision makers can generate collective behaviors that seem, *prima facie*, irrational (e.g. financial bubbles, spread of false information, fashion cycles and fades...).

Simple game with information Cascades

Basic setup:

- Urn A = (2 red balls and 1 blue ball)
- Urn B = (1 red ball and 2 blue balls)
- Nature P_0 and six players $(P_1, ..., P_6)$
- P_0 chooses A with probability $\frac{1}{2}$ and B with probability $\frac{1}{2}$ but does not reveal her choice
- Each of the six players receives a payoff of 1 if they choose the urn chosen by Player 0.

Order of moves:

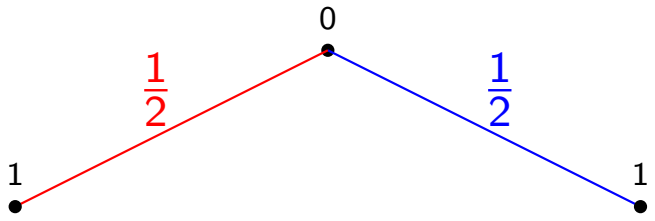
1. P_1 draws a ball from the urn chosen by P0 and chooses an urn;
2. P_2 observes P_1's choice, draws a ball from the urn chosen by P_0, and chooses an urn.
3. P_3 observes P_1 and P_2's choices, draws a ball from the urn chosen by P_0, and chooses an urn

...

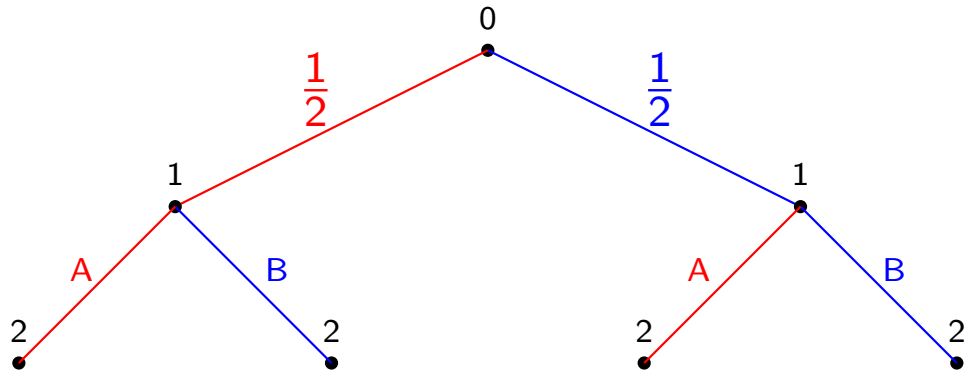
The Extensive Form Representation with three players (without showing ball-drawing)



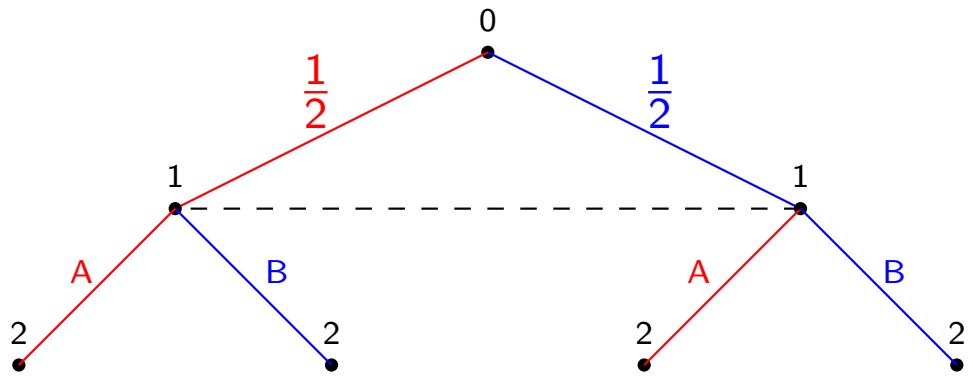
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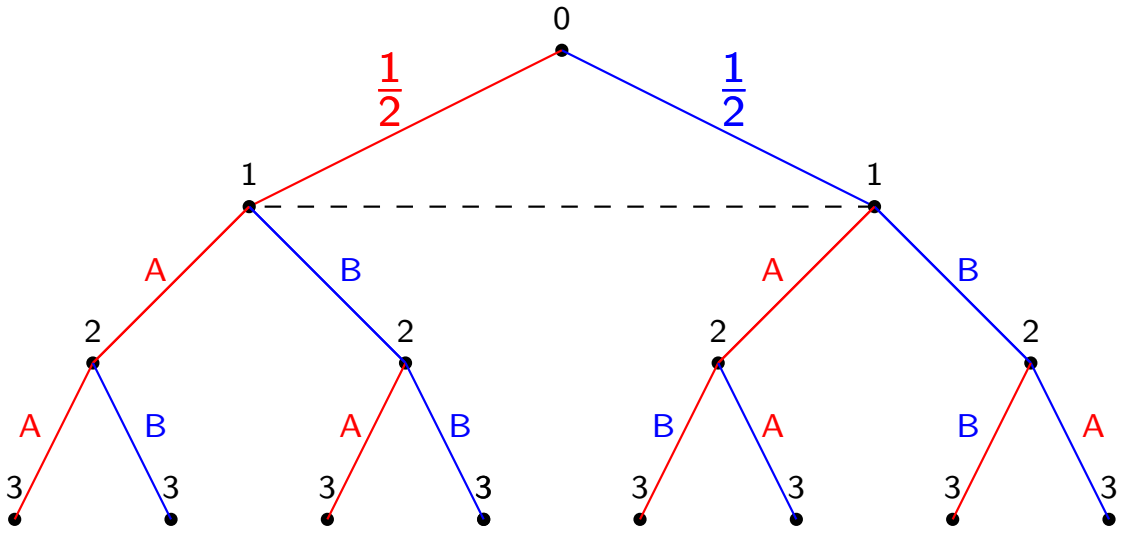
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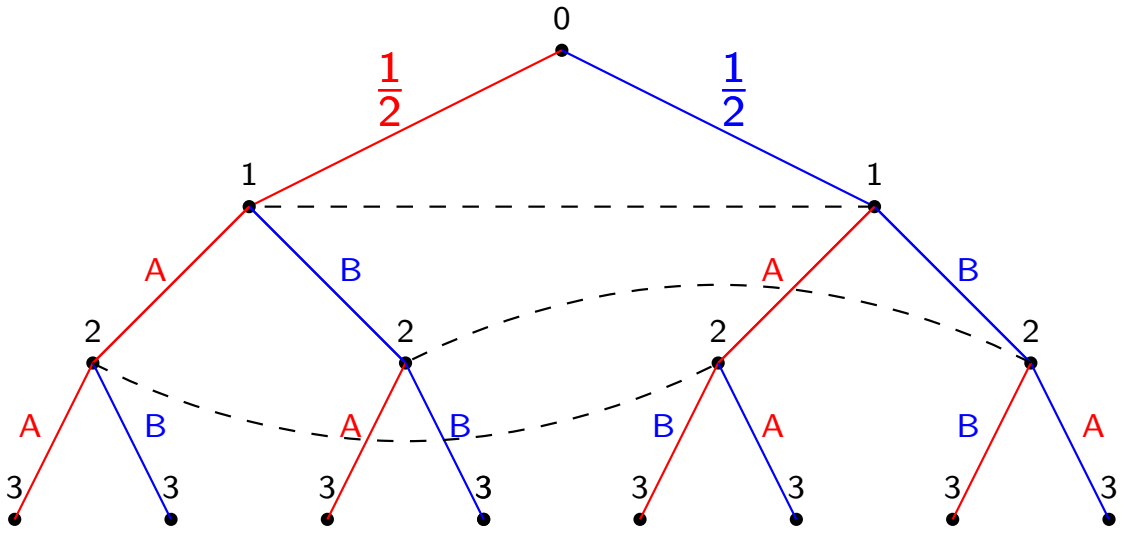
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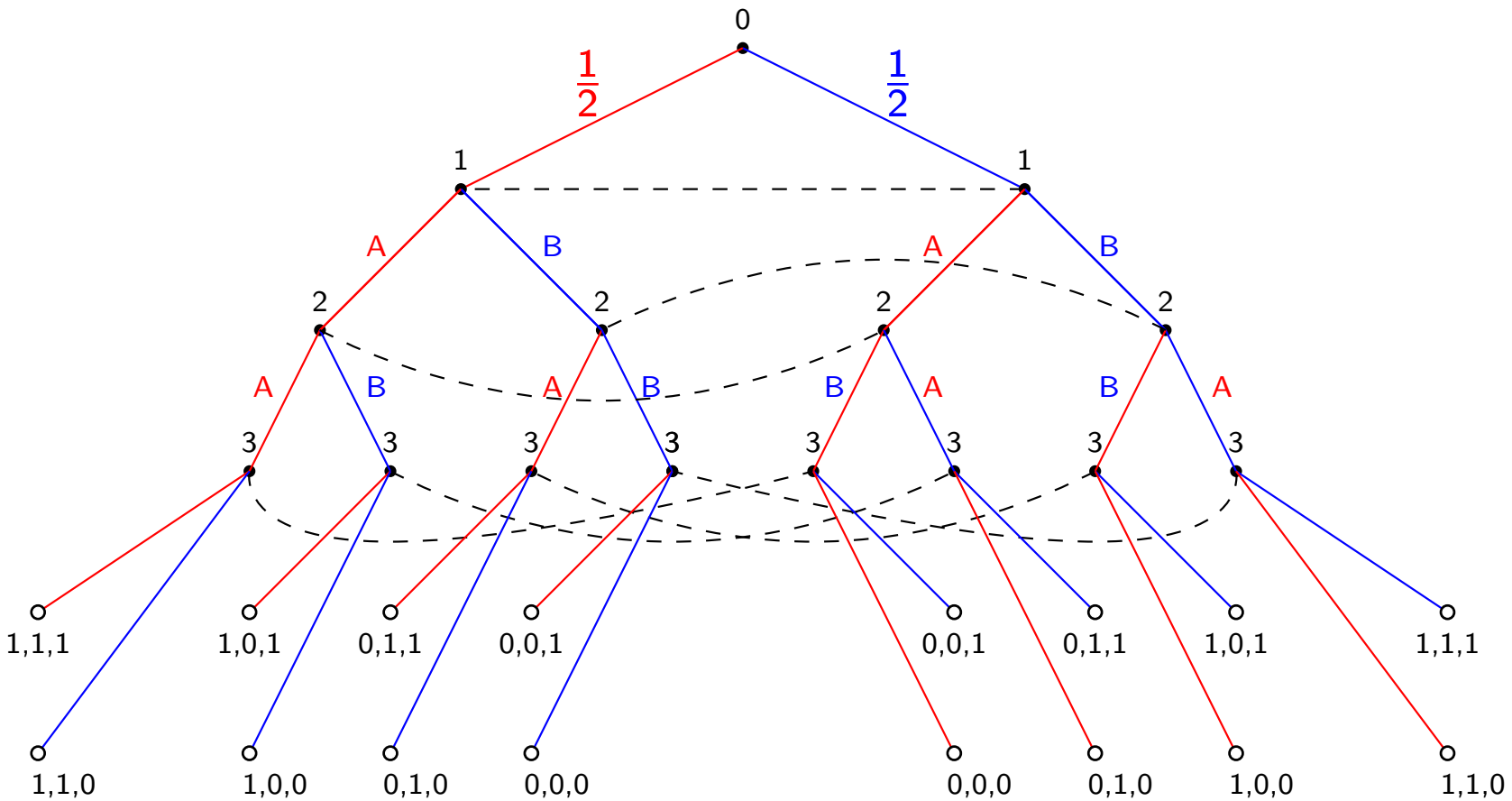
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Intuition for the information cascade

If P_1 and P_2 choose A, then P_3 chooses A, no matter what signal she receives.

1. P_1 chooses A only if she draws a red ball $\Rightarrow$ P_1 received a red ball.

2. If P_2 also draws a red ball (a), then she chooses A for sure; if she draws a blue ball (b) then she is indifferent and chooses A with probability $p \in [0, 1]$.

   P_2 chose $A \Rightarrow$ probability that she drew a red ball (a) is $1/(1+p)$.

3. P_1 and P_2 chose $A \Rightarrow$ both players drew a red ball with probability $1/(1+p)$
   P_1 drew a red ball (a) and P_2 a blue ball (b) with probability $p/(1+p)$.

4. P_3 gets signal b and in total has $1 + 1/(1+p)$ signals indicating the urn is A and $1 + p/(1+p)$ signals indicating that the right urn is B.

5. Thus, if $p < 1$, she should choose A.

Implication: All signals but the first two are ignored – highly inefficient

Does Bayesian Nash equilibrium predict subjects' behavior in Lab?

Anderson and Holt (1997) tested the predicted information cascade experimentally with 6 players.

They conclude that

- "Individuals generally used information efficiently and followed the decisions of others when it was rational."
- "However, some errors, which tended to make subjects rely more on their own private information."

Questions

- Can we explain the deviations from Nash equilibrium with Quantal Response Equilibrium (AQRE)?
- Or should we explain the data with violation of Rational Expectations? I.e. that the subjects do not weigh the sources of information symmetrically
- Do subjects acknowledge their opponents' rationality; or do they systematically overweigh their own information relative to the information they could infer from others' behavior?

Self-Correcting Information Cascades
Goeree, Palfrey, Rogers, and McKelvey (2007)

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Pure cascades and crazes

- Once started, a pure cascade lasts until the end of the game, e.g. BABAAAAAAAAA...AA
- A craze is an interrupted cascade, e.g. BAAAABBA

Bayesian Nash equilibrium is not compatible with crazes, while AQRE is compatible with some crazes.

- BNE predicts that all subjects should have the same behavior as soon as the signals are sufficiently clear.
- AQRE allows for (few) random deviations

Goeree, Palfrey, Rogers, and McKelvey (2007) test the adequacy of AQRE in this context

The experiment

Same design as Anderson and Holt (1997), except that

- To observe some crazes they consider longer sequences with $T = 20$ or $T = 40$ subjects
- In Anderson and Holt, there were only 6 subjects per game
- Goeree et al. allow for variety in signal precision: q is either $5/9$ or $2/3$, while in Anderson and Holt it was fixed at $2/3$.

Violation of Bayesian Nash equilibrium

	q = 5/9 T = 20	q = 5/9 T = 40	q = 6/9 T = 20	q = 6/9 T = 40
# sequences	116	56	90	60
# sequences with pure cascades	5	0	12	8
# sequences without cascades	0	0	0	0
# sequences with broken cascades	111	56	78	52

Table 1. *Percentages of (broken) cascades in our data.*

Prediction of AQRE

Each individual t privately observes a payoff disturbance for each choice, denoted ϵ_t^A and ϵ_t^B, where the $\epsilon_t = \epsilon_t^A - \epsilon_t^B$ is assumed to obey a *Logistic distribution* with parameter λ.

Expected payoffs are:

$$\mathbb{E}(A|p_t, s_t) = \pi_t^{s_t}(p_t) + \epsilon_t^A \text{ and } \mathbb{E}(B|p_t, s_t) = 1 - \pi_t^{s_t}(p_t) + \epsilon_t^B$$

and, therefore the probability of choosing A given p_t and s_t is

$$\begin{aligned}\mathbb{P}(\mathbb{E}(A|p_t, s_t) - \mathbb{E}(B|p_t, s_t) > 0) &= \mathbb{P}(\epsilon_t > 1 - 2\pi_t^{s_t}(p_t)) \\ &= \frac{1}{1 + exp(\lambda \cdot (1 - 2\pi_t^{s_t}(p_t)))}\end{aligned}$$

In AQRE, all actions are chosen with positive probability, and thus

1. The probability that a cascade breaks is increasing with t (there are more chances for breaks);
2. The number of cascades is increasing in T (there are more chances for restarts of cascades).

In AQRE, strategies that are more profitable are more likely to be chosen, implying that DM is predicted to respond to the informativeness of their signal.

1. The probability that a pure cascade breaks decreases in q.
2. Incorrect cascades are shorter than correct cascades and reverse more frequently.
3. Later cascades are more likely to be correct than earlier ones.
4. Reverse cascades are usually self-correcting, and repeat cascades are usually correct.
5. More informative signal lead to faster learning. That is, the rate of increase in the *ex ante* probability of a correct decision as t increases is increasing in q.
6. The probability of a correct decision is higher for a correct than for an incorrect signal.

Results

- The probability that a cascade breaks is increasing in t
- The probability that a pure cascade breaks decreases in q

TABLE 4

Percentages of pure cascades by treatment

	Our data				AH data			QRE–BRF			
	$q = 5/9$		$q = 6/9$		$q = 6/9$	Nash		$q = 5/9$		$q = 6/9$	
	$T = 20$ $M = 116$	$T = 40$ $M = 56$	$T = 20$ $M = 90$	$T = 40$ $M = 60$	$T = 6$ $M = 45$	$q = 5/9$	$q = 6/9$	$T = 20$	$T = 40$	$T = 20$	$T = 40$
First 6 (%)	36	36	50	53	64	98	99	27	26	44	44
First 10 (%)	15	14	31	42		100	100	9	11	28	27
First 20 (%)	4	2	13	32		100	100	1	1	15	14
First 40 (%)		0		13		100	100		0		7

TABLE 5

Percentages of temporary cascades by treatment

	Our data				AH data			QRE–BRF			
	$q = 5/9$		$q = 6/9$		$q = 6/9$	Nash		$q = 5/9$		$q = 6/9$	
	$T = 20$ $M = 116$	$T = 40$ $M = 56$	$T = 20$ $M = 90$	$T = 40$ $M = 60$	$T = 6$ $M = 45$	$q = 5/9$	$q = 6/9$	$T = 20$	$T = 40$	$T = 20$	$T = 40$
First 6 (%)	58	55	42	38	27	0	0	64	62	50	48
First 10 (%)	84	84	69	55		0	0	88	87	71	71
First 20 (%)	96	98	87	68		0	0	99	99	85	86
First 40 (%)		100		87		0	0		100		93

- The number of cascades is increasing in T.

		q = 5/9		q = 6/9	
		N = 20	N = 40	N = 20	N = 40
		M = 116	M = 56	M = 90	M = 60
average number cascades	Our Data	3.47	7.54	2.99	3.73
	QRE - BRF	3.85	7.31	2.81	4.19
	Nash	1.00	1.00	1.00	1.00
average length cascades	Our Data	2.43	2.00	3.27	7.83
	QRE - BRF	1.59	2.04	3.79	6.50
	Nash	17.32	37.25	17.43	37.44

Table 6. *Number and lengths of cascades by treatment.*

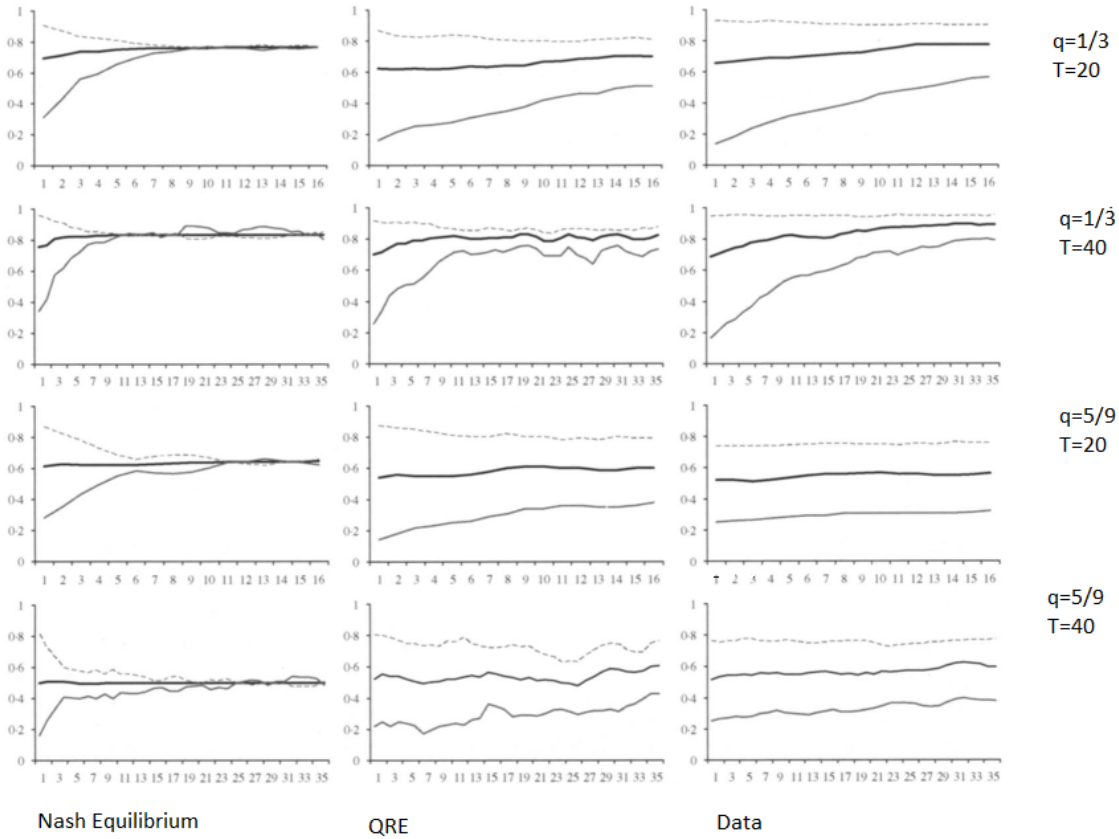
- Later cascades are more likely to be correct than earlier ones
- Reverse cascades are usually self-correcting
- Repeat cascades are usually correct

TABLE 9

Transitions between correct and incorrect cascades in our data

		$T = 20$			$T = 40$		
$q = 5/9$	From\To	Correct	Incorrect	From\To	Correct	Incorrect
	Correct	92·7%	7·3%	Correct	93·6%	6·4%
	Incorrect	22·7%	77·3%	Incorrect	11·0%	89·0%
# Correct	Total = 252	Start = 65	Final = 78	Total = 237	Start = 29	Final = 34
# Incorrect	Total = 151	Start = 51	Final = 38	Total = 185	Start = 27	Final = 22
$q = 6/9$	From\To	Correct	Incorrect	From\To	Correct	Incorrect
	Correct	91·4%	8·6%	Correct	98·7%	1·3%
	Incorrect	30·5%	69·5%	Incorrect	20·0%	80·0%
# Correct	Total = 197	Start = 66	Final = 71	Total = 186	Start = 48	Final = 52
# Incorrect	Total = 72	Start = 24	Final = 19	Total = 38	Start = 12	Final = 8

- More informative signal lead to faster learning. That is, the rate of increase in the *ex ante* probability of a correct decision as t increases is increasing in q.



What feature of the data is not captured with AQRE?

- In AQRE, DM use the same λ to take her decision and to learn from others, implying that they treat all signals symmetrically (Rational Expectations).
- AQRE further implies that indifferent agents should randomize uniformly in their decision.
- Nevertheless, the data show that 85% of the indifferent subjects follow their own signal.
- Besides asymmetric precision, this is reminiscent of the *Base Rate Fallacy* (Kahneman and Tversky, 1973).

Limited Depth of Reasoning and Failure of Cascade Formation in the Laboratory (Kübler and Weizsäcker, 2004)

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The experiment

The same as Anderson and Holt (1997), except that

- They introduce a separate stage for each player at which she is asked whether or not she wants to buy a signal.
- They also introduce treatments in which subjects were given the information who of the previous subjects had obtained a signal.

Motivation

Predictions of PBE

- Only P_1 buys the signal, other player just follow the decision of P_1
- In experiment, this prediction is falsified

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1. They think others are less rational: violating Rational Expectations
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Testing which explanation applies

- Compare with a treatment where participants know if their opponents bought the signal

The Model

- $u(A, s_t) = \pi_t^{s_t} - c$ if they bought the signal; $u(A, s_t) = \pi_t^{s_t}$ otherwise.
- The probability of choosing A is

$$\mathbb{P}(A|s_t, \lambda_1) = \frac{\exp\left(\lambda_1 u(A, s_t)\right)}{\exp\left(\lambda_1 u(A, s_t)\right) + \exp\left(\lambda_1 u(B, s_t)\right)}$$

- Subjects are assumed to anticipate their own decision probabilities of choosing an urn, and to calculate the expected utility of buying a signal

$$\hat{u}(\text{"}buy\text{"}) = \sum_s \mathbb{P}(s_t = s)\mathbb{P}(A|s, \lambda_1)u(A, s).$$

- So the probability of buying a signal is then

$$\mathbb{P}(\text{"buy"} \ |\lambda_1) = \frac{\exp\left(\lambda_1 \hat{u}(\text{"buy"})\right)}{\exp\left(\lambda_1 \hat{u}(\text{"buy"})\right) + exp\left(\lambda_1 \hat{u}(\text{"no buy"})\right)}$$

The Model

- The subject is aware that all other subjects follow the logistic decision process described above,...
- she attributes a possibly different precision parameter to the decisions of her opponents: λ_2 instead of λ_1.

- Thereby, the "rational expectations" assumption made in AQRE can be tested

- Analogously, when a subject considers the reasoning that others apply when thinking about third subjects, we allow for a third parameter λ_3;
- ... and so on until λ_6.

Results

1. Subjects learn from observing their predecessors' decisions but fail to realize that other subjects also learn from observing their respective predecessors
2. Subjects buy even more signals when they can observe whether their opponents bought the signal or not – cannot be explained by distrust.
3. AQRE fits the data after relaxing *Rational Expectations Assumption*.

TABLE 2

Response precisions estimated from the experimental data

Data	Pooled	HC	LC	HCHI	NC
λ_1	10·45 (0·000, 0·000)	11·36 (0·000, 0·139)	8·19 (0·000, 0·941)	12·97 (0·000, 0·000)	10·84 (0·000, 0·000)
λ_2	5·94 (0·000, 0·000)	8·12 (0·000, 0·000)	8·31 (0·000, 0·000)	4·71 (0·000, 0·000)	3·77 (0·000, 0·000)
λ_3	1·65 (0·795, 0·194)	1·29 (0·994, 0·641)	2·44 (0·905, 0·575)	0·00 (1·000, 0·970)	0·00 (1·000, 0·996)
λ_4	0·00 (1·000, 0·968)	0·00 (1·000, 0·975)	0·61 (1·000, 0·908)	—	—
λ_5	—	—	373·32 (1·000, 0·981)	—	—
λ_6	—	—	0.00 (0·996, —)	—	—
*ll**	−2045·9389	−518·1821	−470·2698	−523·7681	−464·9299

Note: Numbers in parentheses are (i) the marginal level of significance for the parameter to be different from zero, and (ii) the marginal level of significance for the parameter to be different from the parameter on the next-higher level.

Do we Follow Others when we Should? A simple Test of Rational Expectations (Weizsäcker, 2010)

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Motivation

- Meta-analysis of Anderson-Holt-like experiments
- Given the deviation from Bayesian Nash Equilibrium and AQRE, to what extend choose subjects the *empirically optimal* action?
- Test of the rational expectations hypothesis

Variables

- $contradict_i = \begin{cases} 1 & \text{if } i \text{ contradicts her own signal} \\ 0 & \text{otherwise} \end{cases}$

  E.g., chooses A with signal $s_i = b$.

- $I_i = (h_i, s_i)$ = participants' information where s_i is the signal received by i and h_i the previous decision before i.

- $S(i)$ = set of individuals having the same information as i, i.e., the j's such that $I_i = I_j$,

- $mean_{pay}|contradict_i = \frac{1}{|S(i)|} \sum_{j \in S(i)} 1(s_j \text{"is wrong"})$

- $prop_contradicts_i = \frac{1}{|S(i)|} \sum_{j \in S(i)} 1(contradict_i = 1)$

Results: Reluctance to contradict one's signal

- **Increasing fitted line:** participants roughly respond to incentives;
- **Average player** contradicts her own signal only if the empirical odds ratio of the own signal being wrong, conditional on all available information, is larger than 2:1, rather than 1:1 as would be implied by rational expectations.
- In situations where the evidence conveyed by others' choice is not very clear (low $mean_{pay}|contradict_i$), the information is not only less valuable, but the participants are disproportionately worse in making the correct inferences.

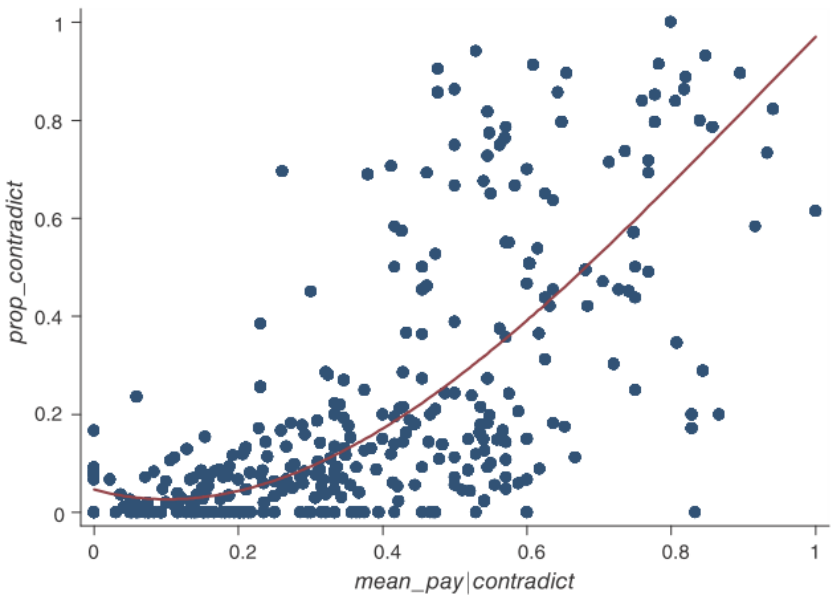


FIGURE 1. PROPORTION OF CONTRADICTING OWN SIGNAL ACROSS ALL OBSERVATIONS WITH $sitcount_i > 10$

Results: Reluctance to contradict one's signal

- Had subjects maximized their payoff in each case, the regression line would be an $S-$shaped line through (0.5,0.5). Here to have the participants contradicting their signal with more than 0.5 probability, the $mean_{pay}|contradict_i$ must be at 0.68.

TABLE 2—FREQUENCY OF CONTRADICTING THE OWN SIGNAL

	Contradict	
	OLS	IV
	(1)	(2)
mean_pay|contradict[1]	0.529***	0.945***
	(0.037)	(0.036)
constant	0.023	−0.135***
	(0.014)	(0.012)
H_0: (0.5, 0.5)	−28.58	−19.38
Observations	21,670	21,670
R^2	0.136	0.052

Notes: Data includes cases with $sitcount_i > 1$. Robust standard errors in parentheses, clustered by $group_i$.

***Significant at the 1 percent level.

Signaling

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Signaling

We focus on Spence's model of education

- Workers have different levels of ability, which is private information and not directly observable by potential employers
- Workers can choose higher education to prove their high ability and get higher wages (signaling)
- Employers can offer wages depending on level of education, in order to incentivize highly productive workers to reveal themselves by pursuing higher education (screening)
- Education itself does not increase ability here, but provides an important signal required for efficient job markets
- Experimental analysis: Kübler et al. (2008)

- Other literature: Brandts and Holt (1992); de Haan et al. (2011); Blume et al. (2020)

Theory: Signaling

One worker and two or three employers (theoretically irrelevant)

1. Nature chooses the worker's ability $a \in \{10, 50\}$ with equal chance.
2. The worker observes a, but employers do not.
3. The worker chooses an education level $s \in \{0, 1\}$ which is observed by the employers.
4. The employers offer a wage $w(s) \in [0, 60]$.
5. The worker picks the employer offering the highest—fair random draw if tie
6. Payoffs are as follows:

$$\pi_{\text{worker}} = w - 450\, s/a$$

$$\pi_{\text{employer}} = \begin{cases} 25 + a - w, & \text{for the employer who hired the worker,} \\ 25, & \text{for the other employer(s).} \end{cases}$$

Equilibrium: Perfect Bayesian Nash equilibrium (strategy σ and beliefs μ)

- As there are more employers than workers, wages will be competitive (wages = expected ability)

Predictions Signaling: Pooling

Both types of workers choose low education,

$$s(\text{low}) = s(\text{high}) = 0 \qquad\qquad w(0) = w(1) = 30$$

- Upon observing $s = 1$ (off-path beliefs) the employer's belief is

$$\mu(a = "high" | s = 1) = \mu(a = "low" | s = 1)$$

- Upon observing $s = 0$ (on path) the employer's belief is

$$\mu(a = "high" | s = 0) = \mu(a = "low" | s = 0)$$

- Regardless s, expected ability $\mathbf{E}[a] = \frac{1}{2}10 + \frac{1}{2}50 = 30$ and empl.'s payoff is

$$25 \geq \begin{cases} 25 & \text{if the employer deviates to } w < 30 \\ 25 + \frac{1}{2}(30 - w) & \text{if the employer deviates to } w > 30 \end{cases}$$

- Since education is costly and irrelevant for wage: low education is optimal

Profits: $\pi_{\text{low}} = \pi_{\text{high}} = 30 - 0 = 30 \qquad \pi_{\text{firm}} = 25$

Separating equilibrium

Only high-ability workers choose high education,

$$s(\text{low}) = 0 \qquad s(\text{high}) = 1 \qquad w(0) = 10 \qquad w(1) = 50,$$

- Upon observing $s = i \in \{1, 2\}$ the employer's belief is

$$\mu(a = \text{"}high\text{"} \,|s = i) = i$$

Profits

$$\pi_{\text{low}} = 10 - 0 = 10 \qquad \pi_{\text{high}} = 50 - 9 = 41 \qquad \pi_{\text{firm}} = 25$$

Cho-Kreps "Intuitive criterion"

Cho and Kreps refine PBE, ruling out some off-path "unreasonable" beliefs in equilibrium.

Intuitive criterion

Take a PBE and an off-path action in this PBE (e.g. $s = 1$ in Pooling). If there is

- a belief where a type (e.g. high ability) would like to deviate
- **no** belief where the other type (e.g. low ability) would like to deviate

We can eliminate the equilibrium.

Pooling is not intuitive: it does not pass the criterion

- with belief supporting separating, the high skilled worker deviate
- with no belief low-skill can benefit from deviating from pooling equilibrium

Separating is intuitive: no off-path action

- there is no off-path action

Theory: Screening

One worker and two or three employers (theoretically irrelevant)

1. Nature chooses the worker's ability $a \in 10, 50$. The worker knows a, but employers do not.
2. The employers each offer two wages, $w(s) \in [0, 60]$; $s \in \{0, 1\}$, which are conditional on the education decision. The worker learns the higher wage for the two contingencies. In the case of a tie, a fair random draw decides whose wage is displayed.
3. The worker chooses an education level $s \in \{0, 1\}$.
4. The employer who offered the higher wage for the education level chosen by the worker hires the worker.
5. Payoffs are as above in the signaling variant.

There is no pooling equilibrium in this game. The unique prediction of the screening game is the separating equilibrium with wage and investment levels as described above.

Experimental design

- Signaling market informed workers move first (choosing education)
- Screening market uninformed employers move first (choosing wages)
- two or three employers $\rightsquigarrow$ Sig2, Sig3, Scr2, Scr3

- Sessions with 9 or 12 subjects, random rematching after periods, with role switching
- 48 rounds = 6 blocks of 8 rounds, during which roles were fixed
- each subject was worker for two blocks and employer for four blocks

- feedback after each round: type and investment decision of worker, wage offers of the employers, profits of all

Results

Table 1
Average investment rates and average wages paid

Treatment	Rounds	Investment rate if worker's type is		Employers' wage paid for the case of	
		low	*high*	*no investment*	*investment*
SIG2	All data	0.10	0.60	14.85	27.89
		(0.04)	(0.08)	(0.30)	(3.35)
	Last block	**0.00**	**0.65**	**13.57**	**29.38**
		(0.00)	(0.19)	(0.26)	(5.52)
SIG3	All data	0.11	0.72	26.01	38.97
		(0.03)	(0.07)	(5.17)	(2.00)
	Last block	**0.08**	**0.83**	**19.84**	**40.89**
		(0.08)	(0.12)	(5.44)	(0.96)
SCR2	All data	0.14	0.68	33.77	43.01
		(0.05)	(0.03)	(2.88)	(1.39)
	Last block	**0.15**	**0.92**	**22.27**	**41.3**
		(0.10)	(0.05)	(4.92)	(2.18)
SCR3	All data	0.07	0.49	30.02	37.62
		(0.00)	(0.20)	(3.59)	(7.96)
	Last block	**0.00**	**0.62**	**27.31**	**43.04**
		(0.00)	(0.26)	(6.24)	(4.08)

Notes. Averages of session averages of investment rates and wages paid. Standard errors of the mean in parentheses. Predictions for the pooling equilibrium: no investment; wage = 30; for the separating equilibrium: only *high* type invests; wage = 10 for *low* type, wage = 50 for *high* type. Last block consists of rounds 41–48 in SIG2 and SCR2 and of rounds 43–48 in SIG3 and SCR3.

- High types of workers invest significantly more often than low types of workers – but separation is imperfect

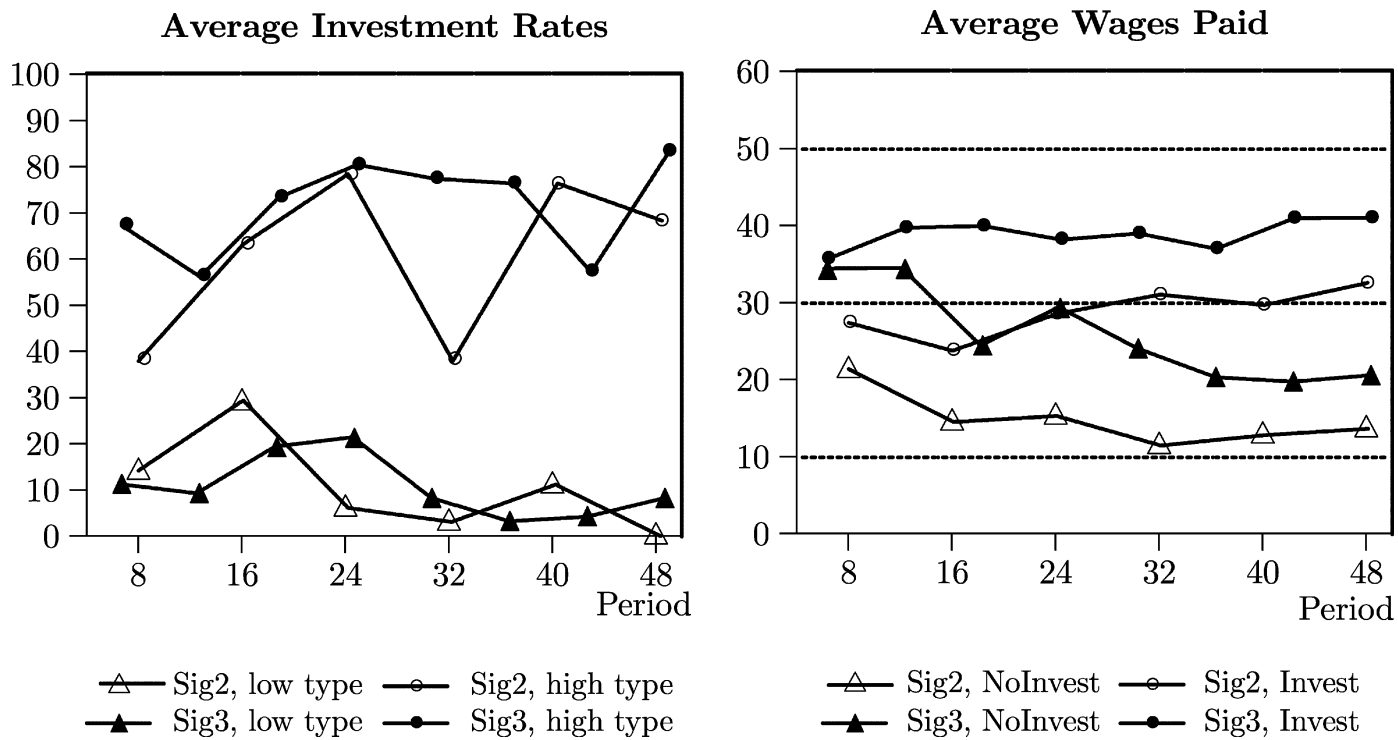


Fig. 1. Evolution of investment rates and wages paid in treatments SIG2 and SIG3.

- Wages offered and wages paid are significantly higher for workers who invest than for workers who do not invest.

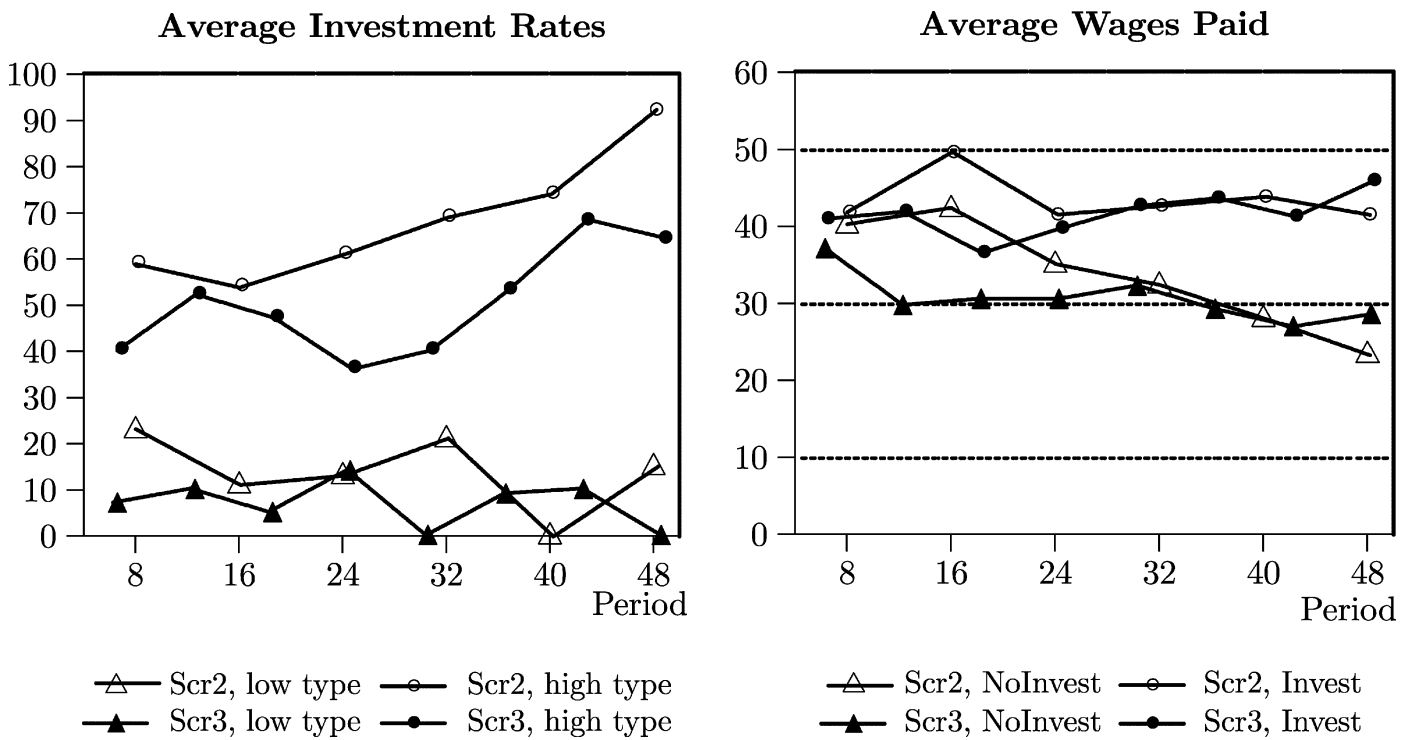


Fig. 2. Evolution of investment rates and wages paid in treatments SCR2 and SCR3.

- Wages offered and wages paid are significantly higher for workers who invest than for workers who do not invest.

Table 3
Average payoffs of workers and average net profits of hiring employers

Treatment	Rounds	Workers' payoff if the type is		Hiring employers' profit after employing a worker who did	
		low	*high*	*not invest*	*invest*
SIG2	All data	11.47	18.1	7.82	16.2
		(2.34)	(2.07)	(1.66)	(2.06)
	Last block	**12.92**	**20.52**	**4.42**	**20.62**
		(0.31)	(3.27)	(3.78)	(5.52)
SIG3	All data	22.51	29.12	−5.54	5.98
		(4.43)	(2.31)	(3.24)	(2.79)
	Last block	**18.39**	**29.73**	**−3.18**	**5.78**
		(6.07)	(2.71)	(7.09)	(2.76)
SCR2	All data	26.49	36.41	−11.57	1.48
		(3.91)	(0.98)	(2.64)	(3.02)
	Last block	**18.74**	**32.37**	**−9.01**	**4.48**
		(6.08)	(1.98)	(3.35)	(3.15)
SCR3	All data	26.88	33.38	−6.33	6.63
		(3.98)	(3.13)	(1.81)	(6.07)
	Last block	**25.50**	**37.49**	**−9.60**	**6.96**
		(5.20)	(3.86)	(4.70)	(4.08)

Notes. Averages of session averages of worker payoff and (hiring) employer profit. Standard deviation of the mean in parentheses. Predictions: in the pooling equilibrium both types earn 30; in the separating equilibrium *high* types earn 41, *low* types earn 10; employers earn zero throughout. Last block consists of rounds 41–48 in SIG2 and SCR2 and of rounds 43–48 in SIG3 and SCR3.

- High-type workers earn significantly higher payoffs than low types.
- Employers' net profits are significantly higher when they employ a worker who has invested than when employing a worker who has not invested.

Table 4
Percentages of outcomes in the last block (in all rounds)

Outcome	SIG2	SIG3	SCR2	SCR3
Separating	53 (42)	54 (38)	43 (37)	39 (41)
Pooling	4 (5)	18 (15)	25 (7)	50 (9)
Other	43 (53)	28 (47)	32 (56)	11 (50)

- Investment rates of high types of workers are below 100%, separation is incomplete – standard result in such experiments
  - logit responses?
- We do not observe wages converging to the levels predicted by the separating equilibrium
  - response to imperfect separation?
- Employers earn profits when employing workers who invested and losses when employing workers who have not invested
  - underestimating separation/rationality of workers?
  - imperfect Bayesian updating?

- Not much of a difference between screening and signaling, or between 2 and 3 employers.

Conclusion

Games of incomplete information

Conclusion

- QRE can be extended straightforwardly to allow for sequential choice, maintaining the standard assumptions of logit choice and rational expectations
- Can capture behavior in some cases reasonably well, but not in general
- One recurrent theme is as in normal-form games: Subjects seem to underestimate the rationality of others
  - Explains deviations from rational-expectation play in cascade formation
  - May explain difficulties of AQRE of capturing behavior in long centipede games
- Subjects seem to even stronger underestimate how others estimate rationality of others
  - Explains why subjects seem to look ahead rather than induce backwards
- Relatedly, subjects do not seem to estimate continuation values in dynamic (bargaining) games – and practically fail to react to variations thereof
  - They do react to variations in outside options, however, suggesting that this "failure" is not about social preferences

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Details on the intuition for information cascades: Notation

- A_i (or B_i) denotes the event "player i chooses A (or B)"
- A sequence of choices is denoted as $A_1 B_2 A_3 A_4$.
- h_t denotes the previous choices observed by P_t; for example $h_3 = A_1 B_2 A_4$.

- s_i (with $s = a$ or b) is the signal received by Player i: If Player i draws a red ball then a_i, otherwise b_i.
- p_t is the posterior probability that P_0 chose A given h_t, so

$$p_t = \mathbb{P}(A_0 | h_t)$$

- $\pi_t^{s_t}$ is the probability P0 chose A, given h_t and s_t, so

$$\pi_t^{s_t} = \mathbb{P}(A_0 | h_t, s_t)$$

Assumption: In case of indifference, a player chooses an urn by a uniform draw (50-50)

Show that if P_1 and P_2 choose A then so does P_3.

Assume P_1 and P_2 chooses A.

1. *Show that $\pi_1^{a_1} = \frac{2}{3}$. Deduce from that P_1 chooses A if an only if she draws a red ball.*
2. *Explain why $\pi_2^{a_2} = \mathbb{P}(A_0|a_1 a_2)$.*
3. *Show that $\pi_2^{a_2} = 1$ and $\pi_2^{b_2} = \frac{1}{2}$. Deduce from that if P_2 draws a red ball then she chooses A for sure and that if she draws a red ball then she chooses A with probability $\frac{1}{2}$.*
4. *Show that if P_1 and P_2 choose A, then the probability that P_2 drew a red ball is $\frac{2}{3}$, i.e. $\mathbb{P}(a_2|A_1 A_2) = \frac{2}{3}$.*
5. *Explain why*

$$\pi_3^{a_3} = \mathbb{P}(a_2|A_1 A_2).\mathbb{P}(A_0|a_1 a_2 a_3) + \mathbb{P}(a_2|A_1 A_2).\mathbb{P}(A_0|a_1 b_2 a_3)$$

and

$$\pi_3^{b_3} = \mathbb{P}(a_2|A_1 A_2).\mathbb{P}(A_0|a_1 a_2 b_3) + \mathbb{P}(a_2|A_1 A_2).\mathbb{P}(A_0|a_1 b_2 b_3)$$

6. *Compute $\pi_3^{a_3}$ and $\pi_3^{b_3}$. Deduce that, whatever the signal she receives, P_3 chooses A when P_1 and P_2 choose A.*

(The solutions of this exercice are given at the end the slides)

Details on the intuition for information cascades: Formal derivation

1. If Player 1 draws a red ball, then, Applying Bayes rule, we have that

$$\pi_1^{a1} = \frac{\frac{2}{3}\cdot\frac{1}{2}}{\frac{2}{3}\cdot\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{2}} = \frac{2}{3}$$

Thus,

$$\mathbb{E}(A_0|a_1) = \pi_1^{a1} > (1 - \pi_1^{a1}) = \mathbb{E}(B_0|a_1)$$

so, P_1 chooses A.

2. Since P_2 believes in P_1's rationality, P_2 deduces from P_1's choice that she drew a red ball. Thus, $\pi_2^{a2} = \mathbb{P}(A_0|A_1 a_2) = \mathbb{P}(a_1|A_1) = \mathbb{P}(A_0|a_1 a_2)$

3. Applying Bayes rule we have

$$\pi_2^{a2} = \mathbb{P}(A_0|a_1 a_2) = \frac{\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{1}{2}}{\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{2}}$$

and

$$\pi_2^{b2} = \mathbb{P}(A|a_1 b_2) = \frac{\frac{1}{3}\cdot\frac{2}{3}\cdot\frac{1}{2}}{\frac{1}{3}\cdot\frac{2}{3}\cdot\frac{1}{2} + \frac{2}{3}\frac{1}{3}\cdot\frac{1}{2}} = \frac{1}{2}$$

Thus

$$\mathbb{E}(A|Aa_2) = \pi_2^{a2} = \frac{4}{5} > \frac{1}{5} = \mathbb{E}(B|A, a)$$

and

$$\mathbb{E}(B_0|A_1 b_1) = \pi_2^{b2} = \frac{1}{2} = \mathbb{E}(B|Ab_2)$$

In case of P_1 and P_2 choosing A, P_3, who believes in P_1 and P_2's rationality, deduces that P_1 drew a red ball for sure and P_2 drew a red ball with probability of $2/3$ or a blue ball with probability of $1/3$. Indeed,

$$\mathbb{P}(a_2|AA) = \frac{\mathbb{P}(AA|a_1, a_2)\mathbb{P}(a_1, a_2)}{\mathbb{P}(AA|a_1a_2)\mathbb{P}(a_1a_2) + \mathbb{P}(AA|a_1b_2)\mathbb{P}(a_1b_2)} = \frac{1 \times \frac{1}{4}}{1 \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4}} = \frac{2}{3}$$

Therefore,

$$\mathbb{E}(A|AAa_3) = \mathbb{P}(a_2|A).\mathbb{P}(A|Aa_3) + (1 - \mathbb{P}(a_2|Aa_2a_3)).\mathbb{P}(U_1|Ab_2a_3) =$$

$$\frac{2}{3}.\frac{(\frac{2}{3})^3.\frac{1}{2}}{(\frac{2}{3})^3.\frac{1}{2} + (\frac{1}{3})^3.\frac{1}{2}} + \frac{1}{3}.\frac{(\frac{1}{3})^3.\frac{1}{2}}{(\frac{2}{3})^3.\frac{1}{2} + (\frac{1}{3})^2.\frac{1}{2}} = \frac{17}{27} > \frac{17}{27} = \mathbb{E}(B|AAa_3)$$

and

$$\mathbb{E}(A|AAb_3) = \mathbb{P}(a_2|AA).\mathbb{P}(U_1|Aa_2b_3) + (1 - \mathbb{P}(a_2|aA)).\mathbb{P}(U_1|r_1, b_2, r_3) =$$

$$\frac{2}{3}.\frac{\frac{1}{3}.(\frac{2}{3})^2.\frac{1}{2}}{\frac{1}{3}.(\frac{2}{3})^2.\frac{1}{2} + (\frac{1}{3})^2.\frac{2}{3}.\frac{1}{2}} + \frac{1}{3}.\frac{\frac{2}{3}.(\frac{1}{3})^2.\frac{1}{2}}{\frac{2}{3}.(\frac{1}{3})^2.\frac{1}{2} + (\frac{2}{3})^2.\frac{1}{3}.\frac{1}{2}} = \frac{5}{9} > \frac{4}{9} = \mathbb{E}(B|AAb_3)$$

So, in any case Player 3 chooses A, even when she draws a blue ball.

Repetition

Games of incomplete information

Repetition questions I

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  - Two urns with red/blue balls, in different compositions, Nature selects an urn, players have to guess the urn Jump
  - Players move sequentially, Nature draws a ball from urn for P_1, who guesses then, everybody sees his guess, then P_2 ...
  - equilibrium: If P_1 and P_2 choose A, then P_3 also chooses A Jump

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- Does AQRE fit the behavior?
  - Goeree et al. (2007) test predictions on long chains (up to 40 subjects) and differing signal precisions
  - Many predictions are confirmed qualitatively Jump
  - In cases of indifference, though, subjects should randomize uniformly, but 85% of supposedly indifferent subjects follow their own signal

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  - An AQRE with asymmetric precisions similar to noisy introspection Jump
  - Each subject has precision λ_1
  - Each subject believes that opponents have precision λ_2
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- What are the results of Weizsäcker/Kuebler (2004)
  - Game where balls have to be bought at small costs; prediction: only P_1 buys, everybody else follows
  - Estimates: $\lambda_1 = 10.5$, $\lambda_2 = 6$, $\lambda_3 = 1.5$, $\lambda_4 = 0$ Jump