Forward induction and psychological games

Summer 26

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Discussion

Topics

- Forward induction: Cooper et al. (1992, 1993); Brandts et al. (1995, 2007); Cachon and Camerer (1996); Huck and Müller (2005); Blume and Gneezy (2010)
- Psychological games: Battigalli and Dufwenberg (2007, 2022); Dufwenberg and Gneezy (2000); Charness and Dufwenberg (2006)

Forward induction

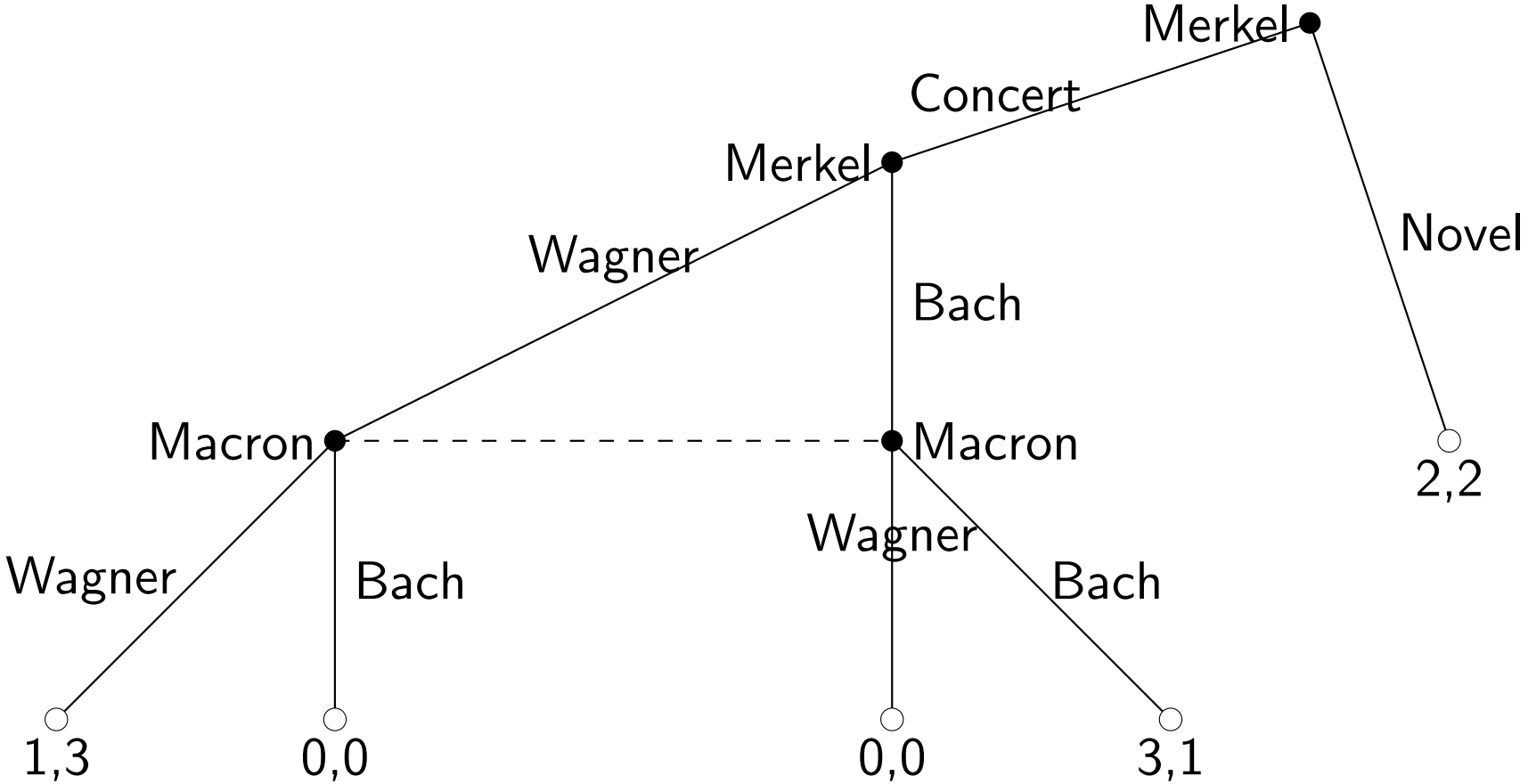
Forward induction and psychological games

Forward induction

- Forward induction All previous actions, even if "unexpected", have been made by rational players
- Backward induction All future actions, regardless of whatever happened before, will be made by rational players

- Literature: Cooper et al. (1992, 1993); Brandts et al. (1995, 2007); Cachon and Camerer (1996); Huck and Müller (2005); Blume and Gneezy (2010)

Example


Merkel
Concert
Merkel
Novel
Wagner
Bach
Macron
Macron
2,2
Wagner
Wagner
Bach
Bach
1,3
0,0
0,0
3,1

Problem

Subgame 1:

	W	B
CW	$3,1$	$0,0$
CB	$0,0$	$1,3$
NW	$2,2$	$2,2$
NB	$2,2$	$2,2$

Subgame 2:

	W	B
W	$3,1$	$0,0$
B	$0,0$	$1,3$

The Subgame-Perfect Equilibria are (CW, W) and (NB, B). Are they both reasonable?

- Imagine that Macron finds himself playing in the second subgame.
- Is it reasonable to believe that Merkel has chosen Bach? Probably not.
- Merkel could have guaranteed herself 2 by playing N. Why would she enter a subgame and play in a manner in which she is only going to get 1?
- Arguably (NB, B) is not reasonable, despite being a subgame perfect equilibrium.

This is an example of forward induction reasoning.

Central to the Forward Induction concept is

- Previous play tells you something about future play
- Subgames cannot be treated in isolation

Thus, forward Induction is not a refinement of SPNE.

Despite its plausibility forward induction has proved tricky to formalize. So, it is beyond the scope of the course.

- Interesting behavioral papers on forward induction: Cooper et al. (1993); Brandts et al. (1995); Cachon and Camerer (1996); Huck and Müller (2005); Brandts et al. (2007); Blume and Gneezy (2010); Blume et al. (2017)

Cooper et al. (1992)

- CG coordination game with two pure Nash Eq
- CG-900 CG where P1 has 900 as outside option
- CG-700 CG where P1 has 700 as outside option

		Column player's strategy 1	Column player's strategy 2
Row player's strategy	1	800,800	800,0
	2	0,800	1000,1000

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- CG-1W CG where P1 can send a non-binding message about the action to be played
- CG-2W CG where both can send a non-binding message about the action to be played

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- CG-1W CG where P1 can send a non-binding message about the action to be played
- CG-2W CG where both can send a non-binding message about the action to be played

Game	Outside option	(1,1)	(2,2)	(1,2) or (2,1)	Total
CG	–	160 (97%)	0 (0%)	5 (3%)	165
CG-900	65	2 (2%)	77 (77%)	21 (21%)	165
CG-700	20	119 (82%)	0 (0%)	26 (18%)	165
CG-1W	–	26 (16%)	88 (53%)	51 (31%)	165
CG-2W	–	0 (0%)	150 (91%)	15 (9%)	165 [b]

- CG almost perfect coord on $(1,1)$; CG-700 bad outside option creates miscoordination and outside-option play; CG-2W works almost perfectly
- CG-900 outside option chosen often (contradicts forward induction), often coordination on $(2,2)$, often also miscoordination

Cooper et al. (1993)

- BOS Battle-of-sexes game with two pure Nash Eq
- BOS-300 BOS where P1 has 300 as outside option
- BOS-300-NF Normal form of BOS-300
- BOS-100 BOS where P1 has 100 as outside option
- BOS-SEQ sequential BOS where Row moves first

		Column Player 1	Column Player 2
Row Player 1		0,0	200,600
Row Player 2		600,200	0,0

FIGURE 1. BATTLE-OF-THE-SEXES GAME (BOS)

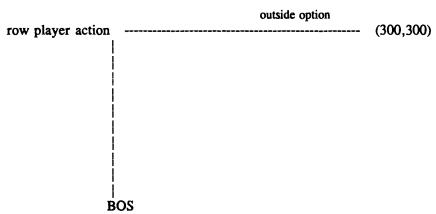


FIGURE 2. BATTLE OF THE SEXES WITH AN OUTSIDE OPTION, BOS-300

		Column Player 1	Column Player 2
Row Player 1		300,300	300,300
Row Player 2		0,0	200,600
Row Player 3		600,200	0,0

FIGURE 3. BOS-300-NF

Cooper et al. (1993)

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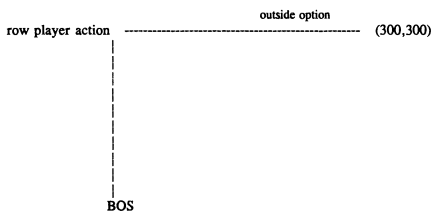


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		Column Player 1	Column Player 2
Row Player	1	300,300	300,300
	2	0,0	200,600
	3	600,200	0,0

FIGURE 3. BOS-300-NF

- Theoretically no difference between BOS and BOS-100
- BOS-300 and BOS-300-NF are dominance solvable, unique $(600, 200)$
- BOS-SEQ with unique SPE outcome $(600, 200)$

TABLE 3—BATTLE-OF-THE-SEXES GAME: FREQUENCY OF OUTCOMES, LAST 11 PERIODS

Game	Outcome			
	(2,1)	(1,2)	DIS	OO
BOS	31 (19)	37 (22)	97 (59)	—
BOS-100	102 (63)	5 (3)	55 (34)	3
BOS-300	119 (90)	0 (0)	13 (10)	33
BOS-SEQ	103 (62)	6 (4)	56 (34)	—
BOS-300-NF	53 (32)	2 (1)	86 (52)	24 (15)

Note: Percentages are shown in parentheses; for BOS-100 and BOS-300, numbers in parentheses show percentage of play in the BOS subgame. For BOS-300-NF, the amounts entered in the OO column correspond to the play of 0 by the row player and 2 by the column player. The (0,1) outcome is aggregated in the DIS column.

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- coordination much better in BOS-100 than in BOS
- high coordination in BOS-300, but outside option frequent
- less coordination in BOS-SEQ – social preferences?
- even less coordination in BOS-300-NF – why?

TABLE 4—BATTLE-OF-THE-SEXES GAME: PERCENTAGE OF STRATEGIES PLAYED BY PLAYER, LAST 11 PERIODS

	Row player			Column player	
Game	Strategy 0	Strategy 1	Strategy 2	Strategy 1	Strategy 2
BOS	—	38 (130)	62 (210)	35 (124)	65 (188)
BOS-100	2 (300)	13 (54)	85 (438)	73 (176)	27 (78)
BOS-300	20 (300)	2 (16)	78 (552)	92 (196)	8 (12)
BOS-SEQ	—	12 (60)	88 (420)	70 (176)	30 (72)
BOS-300-NF	42 (300)	5 (72)	53 (384)	64 (232)	36 (156)

Note: Numbers in parentheses are the expected profits to the player from playing the associated strategy given that his opponent is playing according to the observed frequencies.

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- BOS strategies reasonably close to Nash eq (.25, .75), bias to 50-50 (QRE?)
- BOS-100 coordination much better than predicted, even for P2 – why?
- BOS-300 coordination overall good, but outside option 20%
- BOS-SEQ P2 less willing to coordinate than in BOS-300
- BOS-300-NF coordination worse than BOS-100 for both, 42% choose OO
  P1 rarely chooses the dominated strategy 1, but P2 often choose their weakly dominated 2 – not Level-1 or Level-2, possibly logit equilibrium
- Choice frequencies highly correlate with differences in exp payoffs $\rightsquigarrow$ logit eq

Brandts et al. (1995)

- BOS framed as market entry game
- Outside option: each firm serves their traditional market
- Game: choice between low marketing (R1/C1) and high marketing (R2/C2)

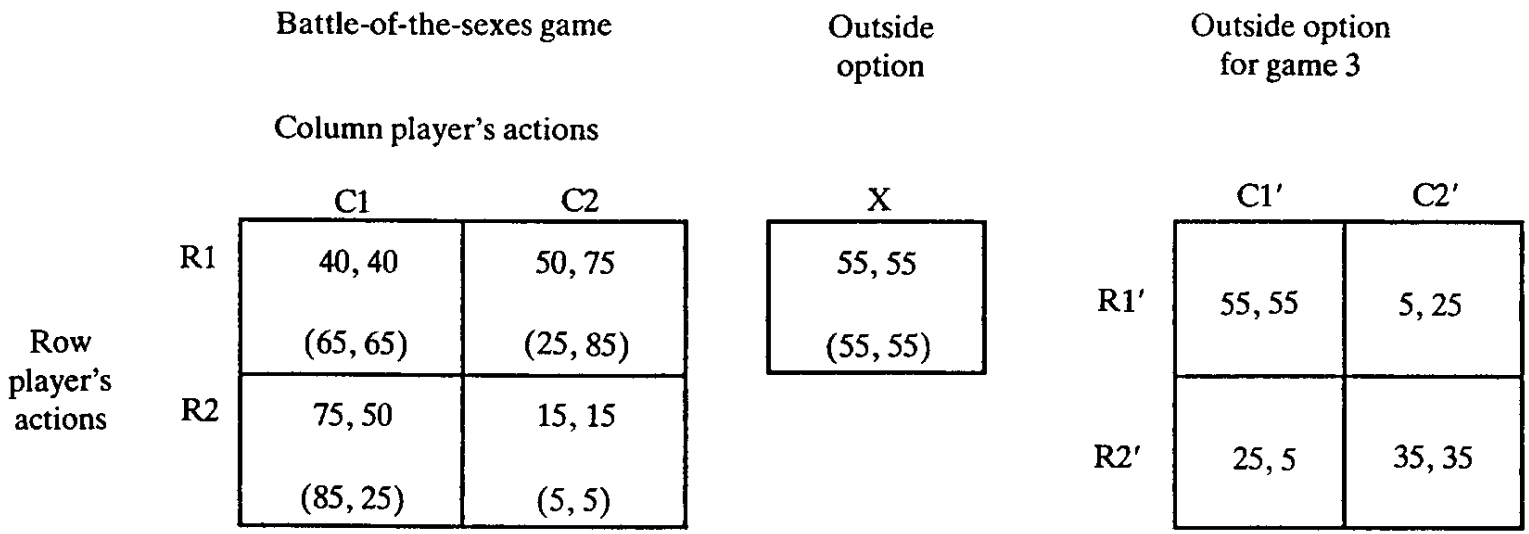


Fig. 1. Games 1–3.

- Experiment 1: payoffs as in top rows of BOS and X $\rightsquigarrow$ dominance solvable
- Experiment 2: payoffs as in bottom rows of BOS and X $\rightsquigarrow$ not dominance solvable
  Note also that the BOS-Nash eq are socially efficient in Exp 1 but not in Exp 2
- Experiment 3: payoffs as in top rows of BOS, but outside option is the dominance solvable game to the right – intuitively same as Experiment 1 (though not formally)

Brandts et al. (1995)

Table 1
Experiment results

	Outcome % if outside option is rejected			Outside option as % of total	Number of observations
	R1/C2	R2/C1	Other		
Experiment 1, part I	31	17	52	[a]	48
Experiment 1, part II	69	0	31	46	48
Experiment 1, part III	91	0	9	31	48
Experiment 2, part I	19	19	63	[a]	32
Experiment 2, part II	29	19	52	34	32
Experiment 2, part III	23	8	69	59	32
Experiment 3, part I	25	13	63	[a]	8[b]
Experiment 3, part II	11	11	78	44	16
Experiment 3, part III	13	0	88	50	16

[a] No outside option was present.
[b] There are only eight observations for the battle-of-the-sexes game since the coordination game on the right-hand side of Fig. 1 was used for two of the four periods in part I.

- Part 1 first four periodes, without OO; Part 2 four periods with OO; Part 3 role reversal, four more periods with OO
- Experiment 1 Part 1 roughly as Nash (50-50 here); then convergence to R1/C2 (forward induction), but again with many choices of OO
- Experiments 2 and 3 more miscoordination than in baseline BOS

Conclusion Forward Induction

- Intuitively very simple, empirically works in simple examples when supported by dominance
- Even when it works, outside option is chosen too often – loss aversion?
- Sometimes, works reasonably well even when not formally implied (BOS-100)
- Sometimes, fails empirically although it should work theoretically (experiment 2, when not supported by dominance) – may depend on whether Nash eq is socially efficient
- Some of the deviations seem consistent with logit responses to expected payoffs – logit equilibrium with loss aversion?

- Cachon and Camerer (1996); Huck and Müller (2005)
  - Paying a fee to play a game (burning money), when optional, has same effect as an outside option (both theoretically and empirically)
  - However, the fee works even when mandatory – loss avoidance
- Further reading: Brandts et al. (2007), Blume and Gneezy (2010)

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Repetition

Forward induction and psychological games

Repetition questions