Ultimatum Games
Inequity Aversion or Reciprocity

Summer 25

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Ultimatum game

Ultimatum game (Güth et al., 1982)

There is a cake worth C Dollar and a game that proceeds in two stages.

1. Player 1 chooses an allocation $(C - x, x)$, $x \geq 0$.
2. Player 2 chooses "accept" or "decline".

If 2 accepts, final payoffs are as proposed, $(C - x, x)$, otherwise both get 0.

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- 2 accepts $\Leftrightarrow$ $x \geq 0$, and 1 proposes $(C, 0)$
- 2 accepts $\Leftrightarrow$ $x > 0$, and 1 proposes $(C - \varepsilon, \varepsilon)$

The First Test (Güth et al., 1982)

- C=4 and C=10.
- Results in Week 1 (inexperienced subjects)
  - All offers at least 1
  - Modal offer 50% (7 out of 21)
  - Mean offer 37%
- One week later (experienced subjects)
  - 20 out of 21 offers at least 1
  - 2 out of 21 offers 50%
  - Mean offer 32%
  - 5 out of 21 offers rejected.

Systematic deviation from the "game theoretic prediction".

Later experiments (Cooper and Kagel, 2013)

Typical experimental design

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Typical results in Western countries for C between \$10 and \$30

- Proposals x are usually around 40%–50% of C
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- Optimal proposals are therefore in the range of 40%–50%

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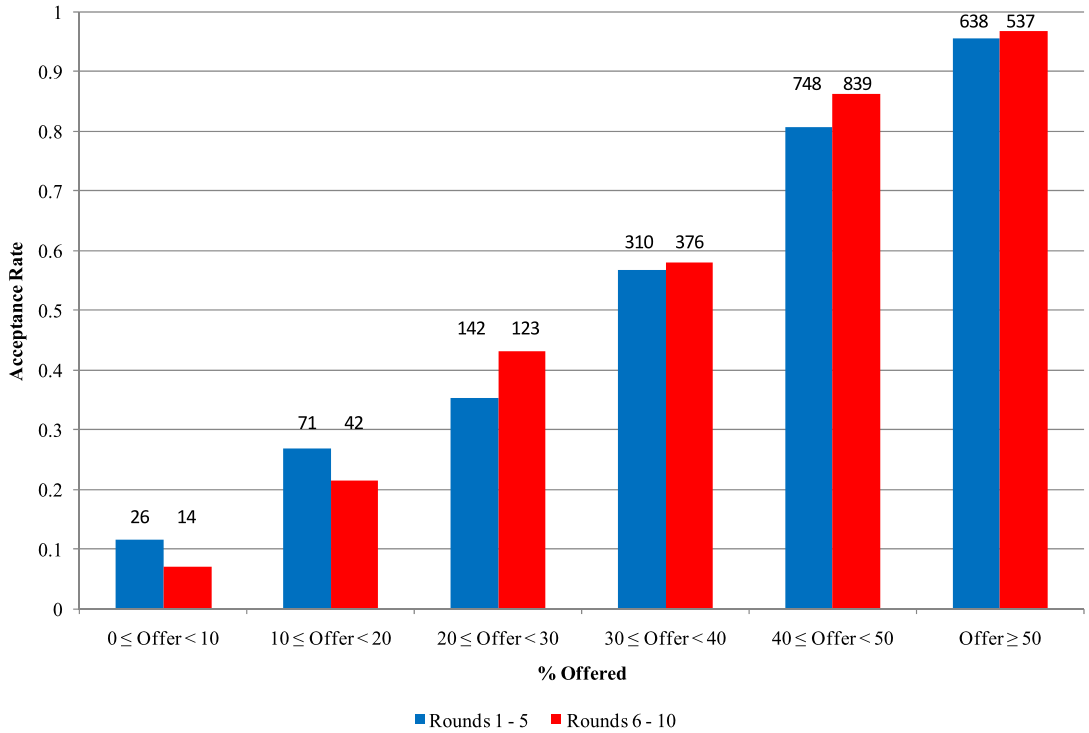
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High stakes

- If C is higher, lower proposals get accepted, too, but rarely below 20%
- In developing countries, \$50 may be monthly incomes, rejection rates decline further
- Median proposal remains at 40%–50% of C, potentially due to risk aversion of proposer

Experience (Cooper and Dutcher, 2011, meta analysis)



- Less acceptance of low offers ($< 20\%$), more acceptance of higher offers
- Effects not strong, optimal proposal continues to be in range 40%–50%
- No convergence to subgame perfection for payoff maximizers
- Increase of acceptance of offers in $[20, 30)$ indicates change in preferences

In small scale societies (Henrich et al., 2001, 2005)

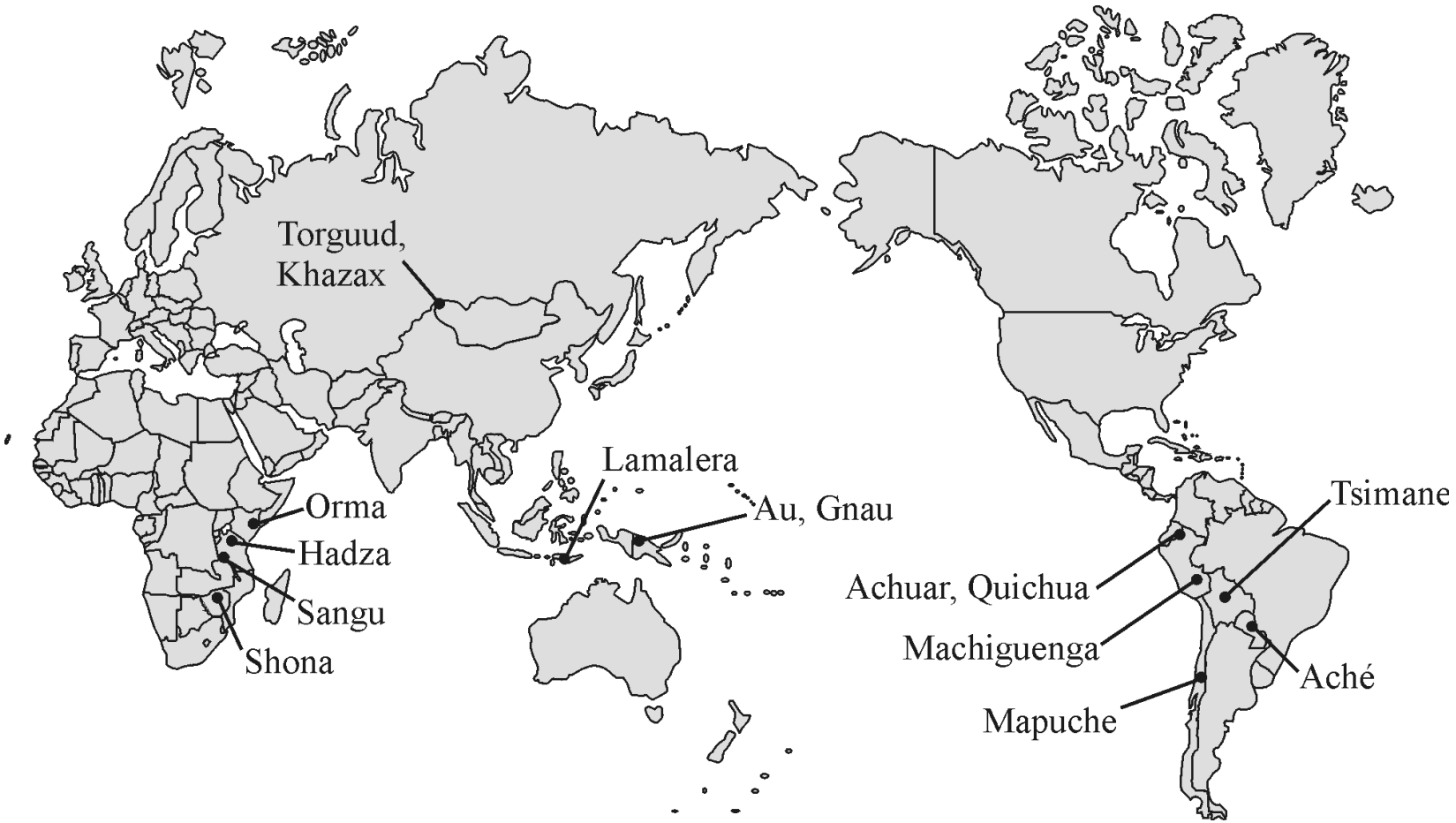


Figure 1. Locations of the 15 small-scale societies.

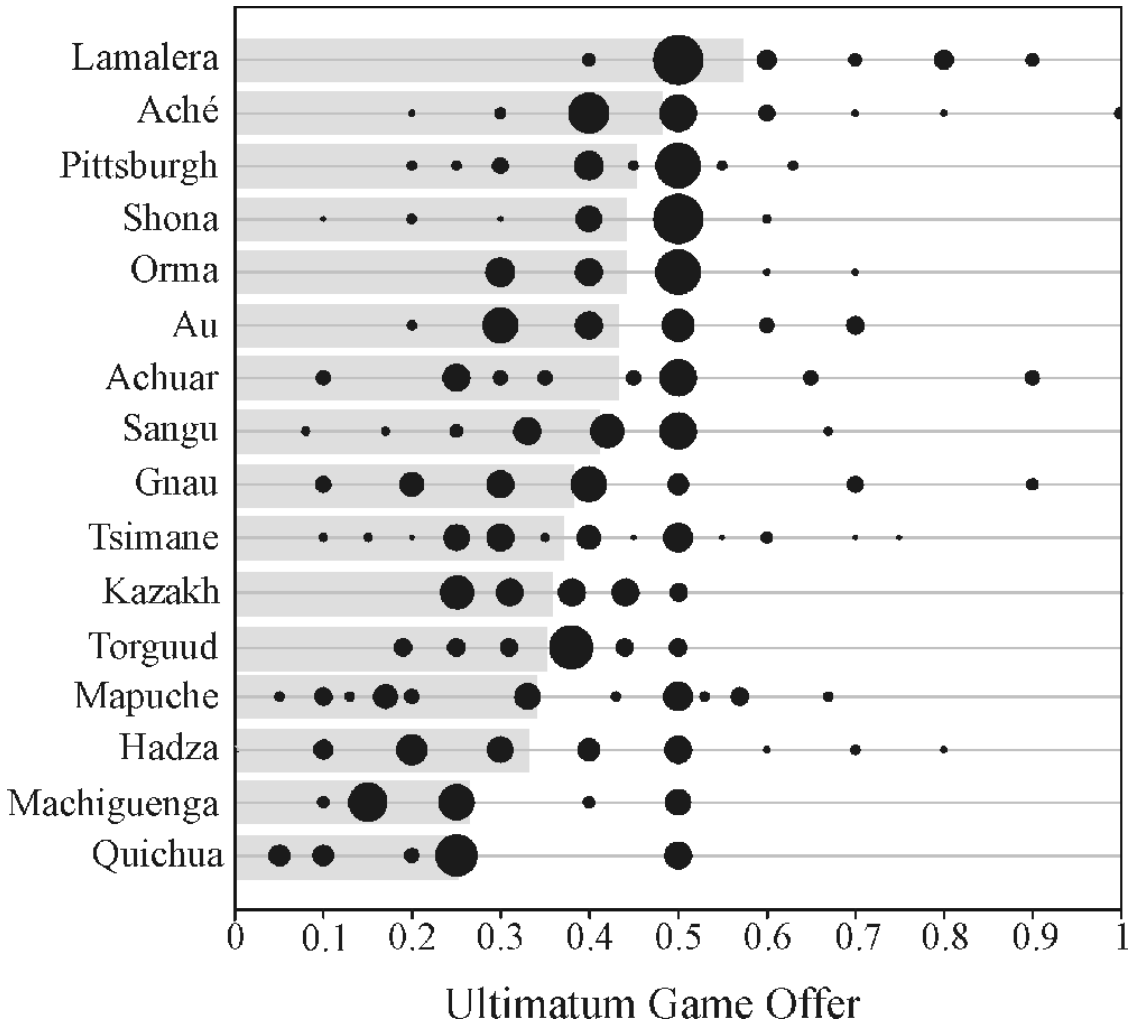
Results

The self-interest model is not supported in any society, and there is considerable variability across societies.

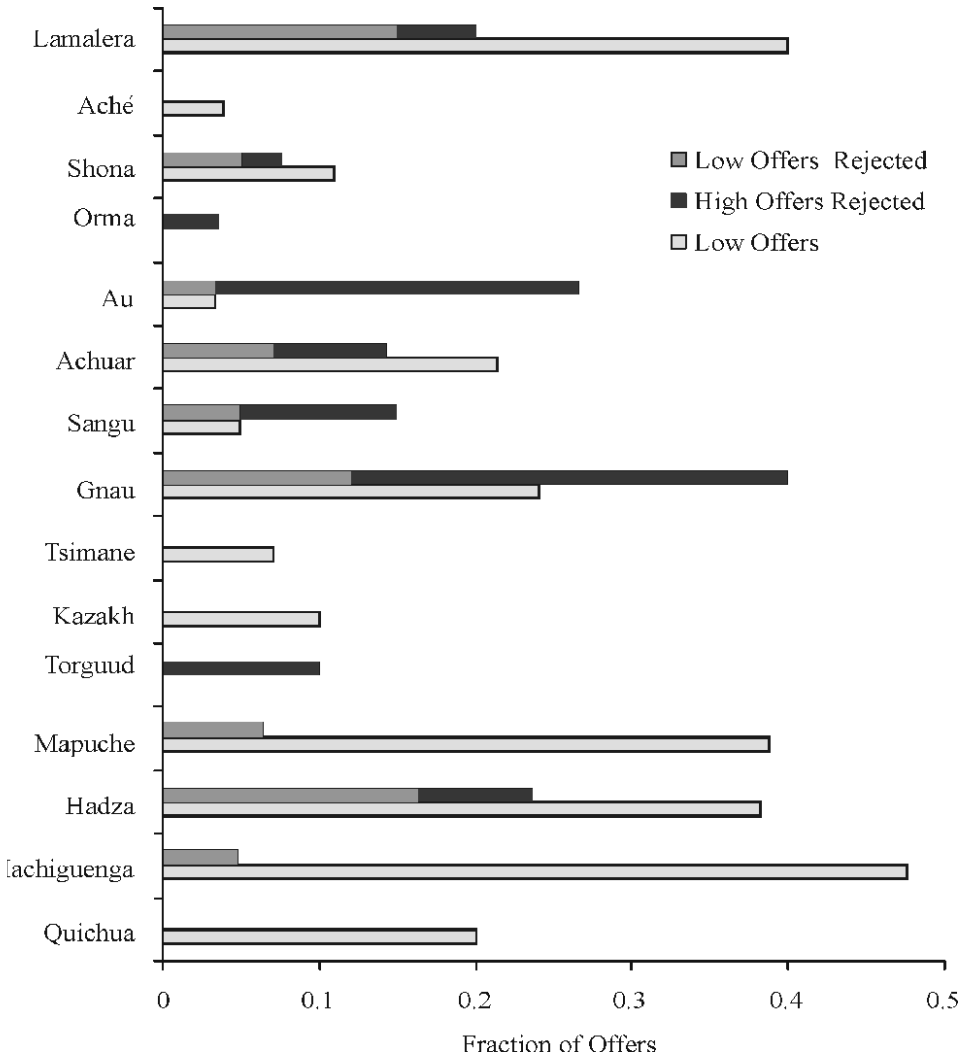
Group differences are mainly explained by between-group variation.

- The degree of market integration:
  - frequency of trade with strangers or non-kin,
  - dependence on buying/selling goods rather than self-production,
  - participation in wage labor or commerce,
  - frequency of interaction with markets,
  - extent of monetization of the economy.
- The potential payoffs to cooperation.
  - In some societies, people can survive largely on their own or within small family units.
  - In others, successful production requires coordinated collective action, so cooperating yields large benefits.

Distribution of Offers (Henrich et al., 2005)



Rejection Behavior (Henrich et al., 2005)



'igure 3. Summary of responder's behavior in ultimatum game. 'he lightly shaded bar represents the fraction of offers that were ess than 20% of the pie. The length of the darker shaded bar gives ne fraction of all ultimatum game offers that were rejected, and

Economic Determinants of Group Differences

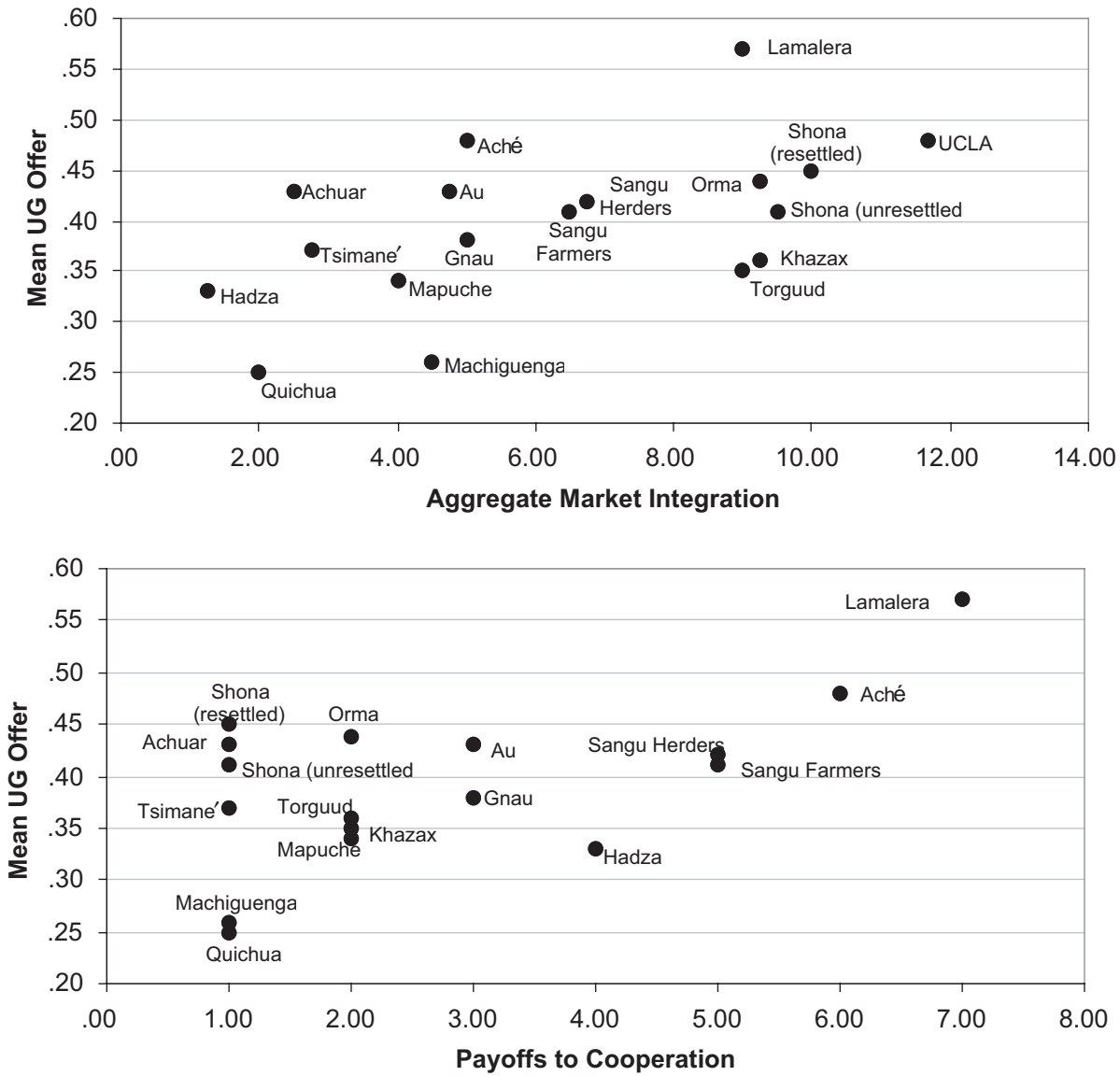


Figure 5. Plots Mean UG offers as a function of the PC and AMI indices. Because AMI and PC are almost uncorrelated ($r = .09$), these bivariate plots give a useful picture of their effects.

Explanations I: Alternative solution concepts

No convergence to SPE, subgame perfection perhaps too strong

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Nash equilibrium Both players choose their strategies simultaneously

- Player 1 picks an offer, player 2 picks a list of offers he would accept (and sticks with that list when the actual offer comes)
- Assumption does not fit all experimental protocols, but NE can explain all allocations and rejections ex post
- Rejections are "misunderstandings" (different equilibria being played)

Inequity Aversion

Ultimatum Games Inequity Aversion or Reciprocity

Explanations II: Alternative utility functions

Subjects generally tend to

- Transfer positive amounts in dictator games $\rightsquigarrow$ altruism
- Reject small yet positive offers in ultimatum games $\rightsquigarrow$ spite/envy

CES utilities with constant altruism weights cannot account for this!

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CES utilities with constant altruism weights cannot account for this!

Inequity aversion (Fehr and Schmidt, 1999)

Let π_1, π_2 denote the payoffs of players 1 and 2, respectively. Utility of $i \neq j$ is

$$u_i(\pi_i, \pi_j) = \begin{cases} \pi_i - \alpha \cdot (\pi_j - \pi_i), & \text{if } \pi_i < \pi_j, \\ \pi_i - \beta \cdot (\pi_i - \pi_j), & \text{if } \pi_i \geq \pi_j, \end{cases}$$

$$= \begin{cases} (1+\alpha)\pi_i - \alpha\pi_j, & \text{if } \pi_i < \pi_j, \\ (1-\beta)\pi_i + \beta\pi_j, & \text{if } \pi_i \geq \pi_j, \end{cases}$$

with $\alpha > \beta > 0$.

Inequity Aversion: Illustration

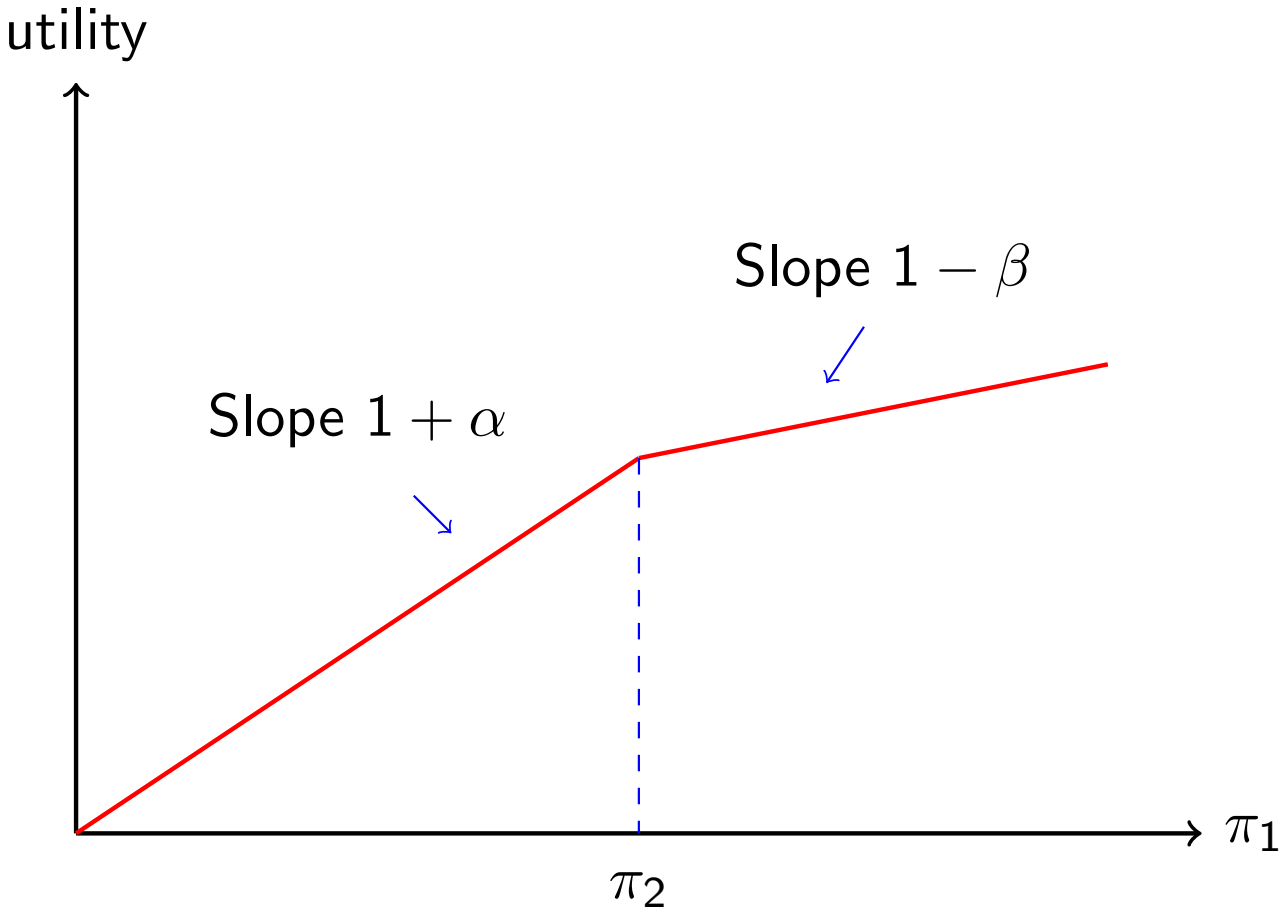
In a 2 players game

$$u_1(\pi_1, \pi_2) = \pi_1 - \alpha \max\{\pi_2 - \pi_1, 0\} - \beta \max\{\pi_1 - \pi_2, 0\}$$

Three parts

1. Standard utility
2. Dislike of having less than the other player
   - $\max\{\pi_2 - \pi_1, 0\} = 0$ if player 1 has more
3. Dislike of having more than the other player
   - $\max\{\pi_1 - \pi_2, 0\} = 0$ if player 2 has more

Inequity Aversion



- Utility function has a kink at π_2
- β assumed to be less than 1

Inequality Aversion and the Ultimatum Game

Assume pie is of size \$10. What will player 2 do if player 1 offers to keep x and give player 2 $(10 - x)$?

Player 2's utility is then

$$u_2(x_1, x_2) = x_2 - \alpha \max\{x_1 - x_2, 0\} - \beta \max\{x_2 - x_1, 0\}$$

Hence, he gets

$$\begin{cases} 0 & \text{if he rejects} \\ 10 - x - \alpha(2x - 10) & \text{if he accepts and } x \geq 10 - x \\ 10 - x - \beta(10 - 2x) & \text{if he accepts and } x \leq 10 - x \end{cases}$$

Player 2

If $x \leq 10 - x$, then player 2 is getting more than player 1,

- his utility can written as

$$(1 - \beta)(10 - x) + \beta x \geq 0$$

$\Longrightarrow$ he always accepts the offer

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If $x \geq 10 - x$, then player 2 is getting less than player 1,

- If he accepts he gets $(1 + \alpha)(10 - x) - \alpha x$
- He rejects such an offer if $x > (1 + \alpha)/(1 + 2\alpha) * 10$
- This is the maximal share of the pie that player 1 can get
  - If $\alpha = 0$ then this is 10
  - As $\alpha \to \infty$, this fraction goes to $1/2 \cdot 10$

What about player 1?

First, notice they can always guarantee themselves a payoff of 5

- Offer $x = 5$
- We know that this is accepted

$\Longrightarrow$ they never make an offer which is rejected.

Their offers lie in $[5,\ 10 * (1+\alpha)/(1+2\alpha)]$, where they get more then player 1

Their utility is

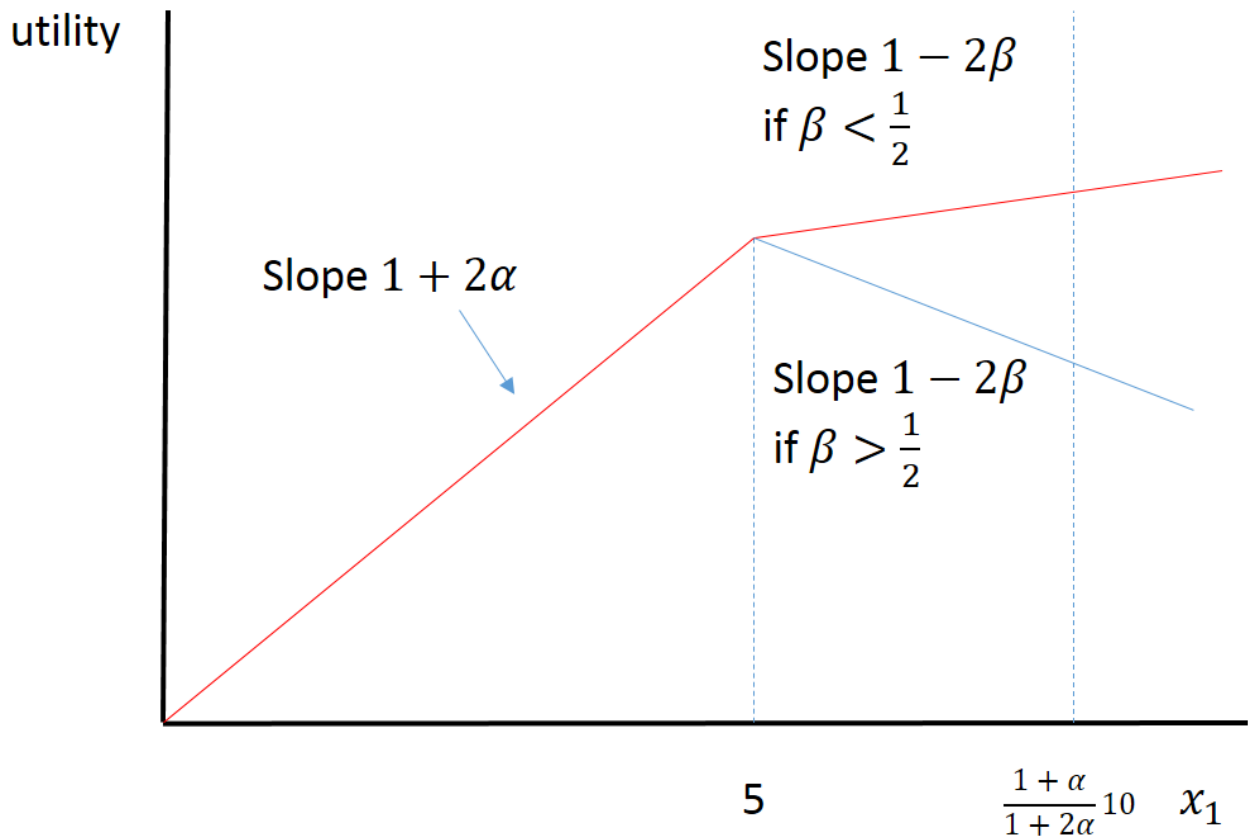
$$u_1(x,(10-x)) = x - \beta(2x-10) \implies \frac{\partial u_1(x_1,x_2)}{\partial x} = 1 - 2\beta$$

- If $\beta < 1/2$, u_1 increases in x, they take the minimal accepted offer:

$$10 * (1+\alpha)/(1+2\alpha)$$

- If $\beta > 1/2$, utility is decreasing in x, they take the 50/50 split

Inequality Aversion



Reciprocity

Ultimatum Games Inequity Aversion or Reciprocity

Fairness

The Fehr-Schmidt model provides one motives for social preferences

- Inequality aversion

However this is not the only possibility, for instance

- Take the standard ultimatum game
- But now restrict the strategy space of player 1 so that the maximum they can offer player 2 is \$2
- How would you respond to an offer of \$2 as player 2?

Experiment : Nelson (2002)

Two versions of the Ultimatum game

- Standard ($20 pie)
- Truncated: max offer $4 ($\rightsquigarrow m = 0.2$)
- All players played both sides of both games
- Behavior of responders extracted using strategy method

The Fehr-Schmidt model predicts that if you rejected $2 in the original game, you must also reject it in this game

- The only thing that matters is outcomes

Experiment: Nelson (2002)

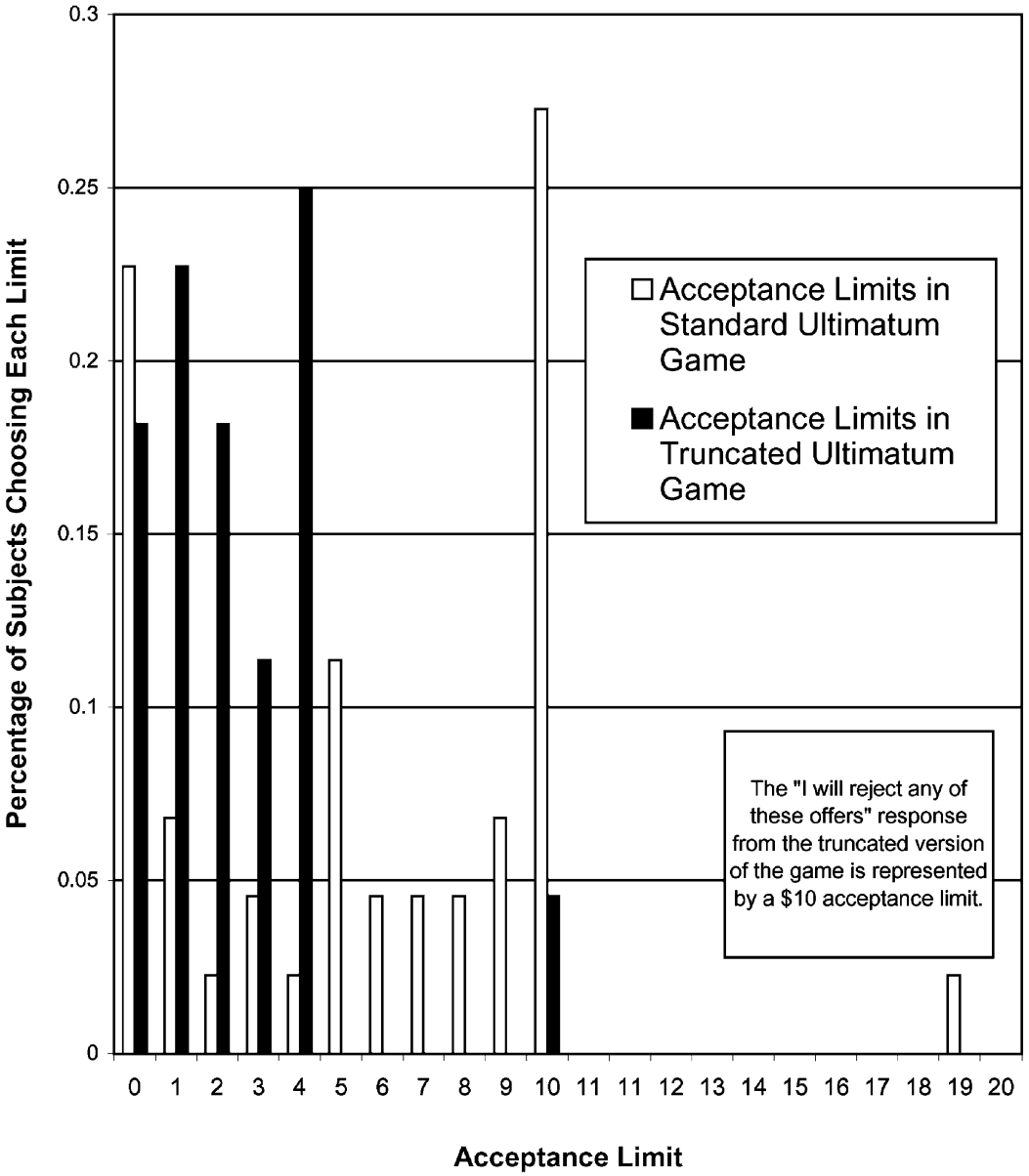


Fig. 1. Cross-treatment acceptance limits.

Fairness

Two key ideas

1. People are willing to sacrifice their own payoff to help those that they think have been kind to them
2. They are prepared to give up their own payoff to punish those that they think have been unkind

$\Longrightarrow$ model of fairness and reciprocity

We need to measure how kind one player is being to another

Fairness

- S_1 be the set of strategies that player 1 can choose from
- S_2 be the set of strategies that player 2 can choose from
- $\pi_i(s_1, s_2)$= material payoff of player i if strategies (s_1, s_2) are played

We want to develop a kindness function

$$f_1(a_1, b_2)$$

How kind does player 1 think they are being if they play a_1, and they think that player 2 will play b_2?

An Example

Player 1's actions	b_2
a_1^1	$3, 9$
a_1^2	$4, 5$
a_1^3	$7, 1$
a_1^4	$-1, -1$

Note

- a_1^1 gives player 2 the highest possible payoff
- a_1^4 is Pareto dominated
- a_1^3 gives player 2 the lowest possible payoff ignoring pareto dominated options

How fair are the four actions in absolute terms? How would you measure fairness?

Fairness Function

$\pi_2^h(b_2)$ be the highest payoff that player 1 could give player 2

- In this example 9

$\pi_2^l(b_2)$ be the lowest payoff amongst pareto efficient points

- In this example 1 after eliminating the Pareto dominated option

Let the **equitable** payoff be given by

$$\pi_2^e(b_2) = \frac{\pi_2^h(b_2) + \pi_2^l(b_2)}{2}$$

- In this example 5

Let $\pi_2^{\min}(b_2)$ be the worst possible outcome for player 2

- in our example -1

Fairness Function

Rabin defines the kindness of player 1 to player 2 as

$$f_1(a_1, b_2) = \begin{cases} \frac{\pi_2(a_1, b_2) - \pi_2^e(b_2)}{\pi_2^h(b_2) - \pi_2^{\min}(b_2)} & \text{if } \pi_2^h(b_2) \neq \pi_2^{\min}(b_2) \\ 0 & \text{otherwise} \end{cases}$$

- Player 1 is being 'kind' if they give player 2 more than the equitable split given what they believe about player 2
- The degree of kindness is scaled by the range of possible outcomes that player 2 could have received.

In our example, a_1^1 would be a kind act, as

$$f_1(a_1, b_2) = (9 - 5)/(9 - (-1)) = 0.4$$

Fairness Function

So we now have a way to capture fairness

What about reciprocity?

- P1 wants to be kind to P2 if they think P2 has treated them kindly
- P1 wants to be nasty to P2 if they think that P2 has treated them badly

We use

$$\overline{f}_2(b_2, c_1) = \begin{cases} \frac{\pi_1(c_1, b_2) - \pi_1^e(c_1)}{\pi_1^h(c_1) - \pi_1^{\min}(c_1)} & \text{if } \pi_1^h(c_1) \neq \pi_1^{\min}(c_1) \\ = 0 & \text{otherwise} \end{cases}$$

to capture P1's beliefs about how kind they think P2 is being to them

- b_2 is the action they think P2 is taking
- c_1 is what P1 thinks P2 thinks P1 is playing (!)

Utility

We can now write down the Rabin fairness utility function

$$u_1(a_1, b_2, c_1) = \pi_1(a_1, b_1) + \overline{f}_2(b_2, c_1) f_1(a_1, b_2)$$

- First bit is standard utility
- Second bit is fairness utility
- Payoff increasing in f_1 if $\overline{f}_2(b_2, c_1) > 0$ (i.e. P2 is being fair)
- Payoff decreasing in f_1 if $\overline{f}_2(b_2, c_1) < 0$ (i.e. P2 is being unfair)

Utility

To predict what happens in the game we need a concept of equilibrium

1. Players are doing the best thing, given their beliefs
2. Their beliefs are correct, given their information

Definition

An equilibrium of a Rabin Fairness game is a set of actions $\{a_1, a_2\}$, first order beliefs $\{b_1, b_2\}$ and second order beliefs $\{c_1, c_2\}$ such that

1. $a_i = \arg\max_{a_i' \in S_i} \; u_i(a_i', b_j, c_i)$ for $i, j \in \{1, 2\}$ and $i \neq j$
2. $a_i = b_i = c_i$ for $i = 1, 2$

Fairness in the Ultimatum Game

P2 always accepts P1's offer if it is the highest offer they can make

- $p > 0$ be the size of the pie
- $m \in [0, 1]$ be the maximum share that player 1 is allowed to offer

We want to check whether it is an equilibrium for P2 to accept m

- Assume that m is being offered and that player 2 will accept
- See if there is any benefit to deviating

Fairness in the Ultimatum Game

$$u_2(A, m, T) = \pi_2(A, m) + \overline{f}_1(m, T) \cdot f_2(A, m)$$

Where

- A=the strategy 'accept'; m= offer of P1
- T= minimal amount P2 accept—what P2 thinks P1 thinks P2 is playing

$\Longrightarrow$ $\overline{f}_1(m, T)$ capture P2's feeling about how kind they think P1 is when offering pm and believing the minimual accepted offer is T

Notice that

- $\overline{f}_1(m, T) \geq 0$ as pm is the most P1 can give P2—hence more than the equitable payoff
- $f_2(A, m) \geq 0$, as accepting gives P1 $(1 - m)p \geq 0$, i.e. weakly more than P1 would get if P2 rejects

Fairness in the Ultimatum Game

Thus we have

$$\begin{aligned} u_2(A, m, T) &= \pi_2(A, m) + \overline{f}_1(m, T)\, f_2(A, m) \\ &\geq \pi_2(A, m) \\ &> 0 \\ &\geq \pi_2(R, m) + \overline{f}_1(m, T)\, f_2(R, m) \\ &= u_2(R, m, T) \end{aligned}$$

Where the last inequality comes from the fact that $f_2(R, m) \leq 0$

Fairness in the Ultimatum Game

- However, the same offer might be rejected if P1 could have offered the whole pie
- Assume
  - $p = 1$
  - $m = 1$
- Assume that receiver would accept any offer greater than $z \geq 0$
- We can show that 0.2 is rejected if the maximum offer is $m = 1$, whatever the value of z
- Hence, $z \leq 0.2$ cannot occur in equilibrium

Fairness in the Ultimatum Game

What is the kindness of offering 0.2 in this situation?

Since z is the minimum accepted offer, z is also the minimal payoff of 2 that is not Pareto dominated

- $\pi_2^h(z) = 1$
- $\pi_2^l(z) = z$
- $\pi_2^e(z) = (1+z)/2$
- $\pi_2^{\min}(z) = 0$

Fairness in the Ultimatum Game

Thus, the kindness of an offer 0.2 is given by

$$\begin{aligned}\overline{f}_1(x,z) &= \frac{\pi_2(x,z) - \pi_2^e(z)}{\pi_2^h(z) - \pi_2^{\min}(z)} \\ &= \frac{0.2 - (1+z)/2}{1} \\ &= 0.2 - (1+z)/2 < 0\end{aligned}$$

Fairness in the Ultimatum Game

Is it better to accept or reject that offer?

- $\pi_1^h(0.2) = 0.8$
- $\pi_1^l(0.2) = 0.8$
- $\pi_1^e(z) = 0.8$
- $\pi_1^{\min}(z) = 0$

Accepting has a fairness of 0, as it is the only non-Pareto dominated option

- $(0.8 - 0.8)/(0.8 - 0) = 0$
- Utility is therefore 0.2

What is the fairness of rejecting?

- $(0 - 0.8)/(0.8 - 0) = 0$
- Fairness is -1

Fairness in the Ultimatum Game

- Utility of rejecting is therefore

$$\pi_1(a_1, b_1) + \overline{f}_2(b_2, c_1)\, f_1(a_1, b_2) = 0 + (-1) \cdot (0.2 - (1 + z)/2)$$

- Rejecting is better if

$$0.2 \leq -0.2 + (1 + z)/2$$
$$\iff \quad 0.8 \leq 1 + z \quad \iff \quad -0.2 \leq z$$

- Which it is $\rightsquigarrow$ a proposal 0.2 should be rejected whatever z is
- What is the minimum accepted offer? It is the smallest z that satisfies

$$z \geq -z + (1 + z)/2 \qquad \Leftrightarrow \qquad 3z \geq 1 \qquad \Leftrightarrow \qquad z \geq 1/3$$

Extremely precise predition due to lack of parameters in model

Inequality Aversion vs Fairness

Is there experimental evidence for such fairness concerns?

Three experiments

- Nelson (2002)
- Sutter (2007)
- Xiao and Bicchieri (2010)

References

Ultimatum Games Inequity Aversion or Reciprocity

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