INTRODUCTION TO ECONOMICS FOR PSYCHOLOGISTS: HISTORY, CONCEPTS, AND PSYCHOLOGICAL FOUNDATIONS

PART 2

Can Pleasure Be Measured? Utility, Preference and Choice from Jevons to Revealed Preference

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Outline

The epistemology of utility
Preference and utility
The axioms of transitivity and completeness
Choice, preference and utility
Cardinality and ordinality
Critiques of rationality and experimental economics

The thread of this part

The marginalists built economics on **utility**, a quantity of pleasure. Their critics asked: *can pleasure be measured?*

The answer reshaped the field: utility becomes a mere **ranking** (preference), then a summary of observed **choices** (revealed preference) — and experiments test whether real choices fit.

1

The epistemology of utility

Can pleasure be measured? The marginalists' critics, their answers, and the behaviourist turn in psychology and philosophy of science.

The critics of the marginalists

Many authors criticised marginalist theory for relying on a utility that, they said, **cannot be measured**.

J. E. Cairnes (1874)

"How are we to measure pleasure? . . . We may take exchange-value as the criterion of utility; and this is the test which Mr Jevons ultimately adopts. . . So that what we come to is — exchange-value depends on utility, and utility is measured and can only be known by exchange-value."

W. S. Jevons (1871) himself

"We have no means of defining and measuring quantities of feeling, like we can measure a mile, or a right angle, or any other physical quantity. . . We can hardly form the conception of a unit of pleasure or pain, so that the numerical expression of quantities of feeling seems to be out of the question."

The marginalists' answers

1. **Jevons:** we may *one day* be able to measure it. Many things were not measurable until methods to measure them were invented.
2. **Jevons and Walras:** the theory does not need a *unit* of utility; it only requires that pleasures can be **compared** (see the Walras–Poincaré correspondence). This anticipates **ordinal utility**.
3. **Jevons and Walras:** utility can be measured *indirectly*, through prices — just as we measure gravity indirectly, through its effects.
4. **Jevons and Walras:** utility can be measured with *money*.

Answers 2 to 4 all move away from measuring a feeling directly, towards what can be **compared** or **observed**.

Behaviourism in psychology

Behaviourism emerged at the start of the 20th century as a reaction against the use of **introspection** (see in particular W. James, *Does "Consciousness" Exist?*, 1904).

J. B. Watson (1913), *Psychology as the Behaviorist Views It*

"Psychology as the behaviorist views it is a purely objective experimental branch of natural science. . . . Introspection forms no essential part of its methods, nor is the scientific value of its data dependent upon the readiness with which they lend themselves to interpretation in terms of consciousness."

Logical empiricism and falsificationism

Logical empiricism (neopositivism)

A philosophical movement developed within the **Vienna Circle**. Most of its members (Carnap, Neurath, . . .) share the idea that science must not rest on **metaphysics**.

Science should be built from

1. **protocol sentences**: observable, non-theoretical statements;
2. **theoretical statements** (laws) that link protocol sentences together.

Karl Popper's falsificationism

A statement is **scientific** if it is **falsifiable**: some possible observation could prove it false.

Both doctrines push economists to ask: what in utility theory can actually be **observed** or **refuted**?

Behaviourism and economics

Ordinal utility

Only **ranks** goods according to the agent's preferences or choices.
$\Rightarrow$ **observable** through behaviour and, as we will see, **falsifiable**.

Cardinal utility

Adds a notion of **intensity**: ranks not only the goods but also the *differences* in utility between them.

- In many contexts, economists came to favour **ordinal** conceptions: in this sense they became "behaviourist" at the time — though not in the sense of psychological behaviourism.
- Economics would rest on a **common-sense (folk) psychology**: individuals with beliefs and desires, who make choices with full freedom of action.

2

Preference and utility

With Pareto, utility stops being primary: it becomes a way of representing pairwise comparisons.

The notion of preference

Out of these debates, an idea emerges with **Pareto**: utility may not be the primary notion.

It derives from a more primitive notion, which only asks the individual to make **pairwise comparisons**: **preferences**.

Strict preference

The individual prefers beer to pretzel:

$$\text{beer} \succ \text{pretzel}$$

Indifference

The individual is indifferent between beer and pretzel:

$$\text{beer} \sim \text{pretzel}$$

Weak preference

Either indifferent, or prefers the beer:

$$\text{beer} \succsim \text{pretzel}$$

In exactly the same way, we can define preferences over **bundles of goods**:

$$(x_1, x_2) \succ (x_1', x_2')$$

means that bundle (x_1, x_2) is preferred to bundle (x_1', x_2'); and so on.

Preference and measurement

A possible definition of a measure

A **measure** on a set X is a function that assigns to each element of X *one and only one* number, so that the elements of X can be ranked relative to one another.

Let X be a set of options and $\succsim$ the agent's preference. We want utility to be a measure u **induced by the observed choices**.

Representation

The utility u **represents** the agent's preference $\succsim$ if, for all $x, y \in X$,

$$x \succ y \iff u(x) > u(y).$$

Does a utility measure exist for *every* behaviour?

Preference and measurement: an example

Example

Take a set of three options $X = \{a, b, c\}$. Can each of the following preferences be represented by a measure?

1. $a \succ c, \quad b \succ c$;
2. $a \succ b, \quad b \succ c, \quad c \succ a$.

How would you change them so that a measure exists?

The fundamental question: what must $\succsim$ look like for it to be interpreted as a measure, i.e. when can it be **represented by a utility function**?

Completeness, transitivity and the representation theorem

Definition: completeness

$\succsim$ on X is **complete** if for all $x, y \in X$: $x \succsim y$ or $y \succsim x$.

Definition: transitivity

$\succsim$ on X is **transitive** if for all $x, y, z \in X$: if $x \succsim y$ and $y \succsim z$, then $x \succsim z$.

Theorem (X finite)

A preference relation $\succsim$ is **complete and transitive** if and only if there exists a function u on X that **represents** $\succsim$, i.e. such that for all $x, y \in X$

$$x \succ y \iff u(x) > u(y).$$

Moreover, u is **unique up to an increasing transformation**: u and v both represent $\succsim$ if and only if $u = \varphi \circ v$ for some strictly increasing φ.

Proof of the theorem

Necessity is obvious. **Sufficiency:** by induction on the number of options $|X|$.

Step 1. Let $|X| = 2$, i.e. $X = \{x, y\}$. By completeness, either $x \succ y$ (set $u(x) = 1, u(y) = 0$), or $y \succ x$ (set $u(x) = 0, u(y) = 1$), or $x \sim y$ (set $u(x) = u(y) = 0$). So the theorem holds when $|X| = 2$.

Step 2. Assume the theorem holds whenever $|X| = n - 1$, and take X with $|X| = n$. Pick $x \in X$. By assumption, there exists $v : X \setminus \{x\} \to \mathbb{R}$ such that for all $y, z \in X \setminus \{x\}$

$$y \succ z \iff v(y) > v(z). \tag{1}$$

If $x \sim y$ for some $y \neq x$, set $u(x) = v(y)$. Otherwise, let $X^{\uparrow} = \{y : y \succ x\}$ and $X^{\downarrow} = \{y : x \succ y\}$. Take $y \in X^{\uparrow}$ and $z \in X^{\downarrow}$: then $y \succ z$. Indeed, if by contradiction $z \succsim y$, we would have $x \prec y \precsim z \prec x$, which contradicts the **transitivity** of $\succsim$. Hence, by (1),

$$v^{\uparrow} := \min_{y \in X^{\uparrow}} v(y) \; > \; v^{\downarrow} := \max_{y \in X^{\downarrow}} v(y).$$

Define $u : X \to \mathbb{R}$ by

$$u(y) = \begin{cases} v(y) & y \neq x \\ \frac{1}{2}\left(v^{\uparrow} + v^{\downarrow}\right) & y = x \end{cases}$$

(if $X^{\uparrow}$ or $X^{\downarrow}$ is empty, put $u(x)$ above the maximum or below the minimum of v). u is well defined by the **completeness** of $\succsim$, and it is easy to check that it represents $\succsim$. □

Proof: worked out on the board

(space for the proof worked out in class)

3

The axioms of transitivity and completeness

What an axiom is, why economists like them, and whether these two deserve our trust.

The notion of axiom

Axioms

Axioms are statements, often expressed mathematically,

- that we take to be true — or at least appealing;
- that serve to prove things we did not necessarily take to be true.

Example

Transitivity and completeness are two axioms. Together they let us prove the **existence of an ordinal utility function** that reproduces the agent's choices.

From the 20th century on, axioms become central in mathematics and logic, and, around the 1930s, in **economics**.

Axioms in economics

The advantages of building a theory on axioms:

Semantic

A theory is made of several ideas that are sometimes hard to separate. Stating axioms distinguishes these ideas (**one axiom = one idea**), and so gives a deeper understanding of the theory.

Empirical

Axioms can also delimit which ideas in a theory can be **tested empirically**.

In economics, both advantages play an important role.

Axioms and falsifiability

To make **falsifiable** the existence of an ordinal utility function that the agent maximises, **Paul Samuelson** proposes the **weak axiom of revealed preference**.

WARP

This axiom is usually called **WARP**: the *Weak Axiom of Revealed Preference*. It is stated in terms of *choices*, which can be observed (Section 4).

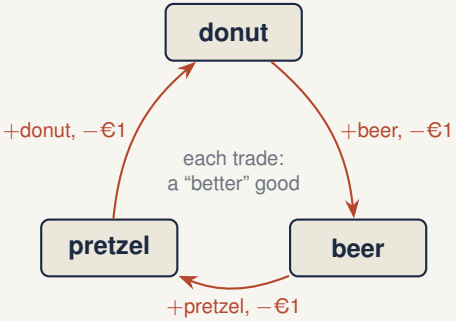
Is transitivity convincing? The money pump

One argument for transitivity rests on the thought experiment of the **money pump** (Ramsey, 1931; Davidson, McKinsey and Suppes, 1955).
Suppose you are **not transitive**: for three goods, beer $\succ$ donut $\succ$ pretzel $\succ$ beer. Say you have €10 and only a donut.

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1. You prefer beer to donut: I sell you some beer for your donut and, say, €1…
2. You prefer pretzel to beer: I sell you some pretzel for the beer I sold you and €1…
3. You prefer donut to pretzel: I sell you some donut for the pretzel I sold you and €1…
4. and so on, until you have **no money left**.



Is completeness convincing?

Completeness presupposes that I can compare *everything* — that nothing is **incommensurable**. Sometimes, however:

- I do not pay attention to some objects (or cannot even imagine them), and so I cannot compare them;
- I **do not want** to compare them: in Wajdi Mouawad's play *Incendies* (*Scorched*), a mother is asked which of her sons must die.

In neither case is there anything **irrational**.

Another problem

Completeness is not really **falsifiable**.

4

Choice, preference and utility

Samuelson’s move: start from what we observe — choices — and ask when they can be explained by maximising a utility.

The notion of choice

In most situations we do not observe the agent choosing between *two* bundles; we observe her choosing **one option from a set** of options — e.g. in a supermarket.

Choice function

Let X be a set of options and 2^X the set of (non-empty) subsets of X, the **menus**. A **choice function** $C : 2^X \to X$ says, for every menu $A \in 2^X$ available to the agent, which option $C(A) \in A$ she chooses.

The big question: under what condition is there a utility function $u : X \to \mathbb{R}$ such that the option chosen in A is the one that gives the agent the most utility in A:

$$C(A) = \arg\max_{x \in A} u(x) \ ?$$

The weak axiom of revealed preference

Definition (WARP)

A choice function C on 2^X satisfies the **weak axiom of revealed preference** if for all menus A, B and all options $x \neq y$:

$$x, y \in A \cap B \text{ and } C(A) = x \implies C(B) \neq y.$$

Once x has been chosen over y, y is never chosen when x is available.

Theorem (X finite)

The following three statements are equivalent:

1. C satisfies **WARP**;
2. there exists a **complete and transitive preference** $\succsim$ such that for every A, $C(A) = \max(A, \succsim)$, the $\succsim$-best option in A;
3. there exists $u : X \to \mathbb{R}$ such that for every A, $C(A) = \arg\max_{x \in A} u(x)$.

WARP with prices

Axiom

If the consumer chooses to buy a bundle x when she could also afford another bundle x', then she should **never** choose x' when she can afford x.

Formally: if x is bought at prices p and $p \cdot x' \leq p \cdot x$ (with $x' \neq x$), then whenever x' is bought at prices p', we must have $p' \cdot x > p' \cdot x'$.

Proof: worked out on the board

(space for the proof worked out in class)

What WARP means (1): no sensitivity to context

WARP implies that the agent is **not sensitive to the context** she is in. Intuitively, this is sometimes a problem.

Amartya Sen's tea party

Imagine we are having tea at my place and I offer you a choice between

- R = "go home" or T = "have another cup of tea": you take the tea, because I am good company;
- R, T or K = "have a little cocaine": now you may well want to leave, because the third option tells you something about the **context** of our meeting.

This example contradicts WARP: $C(\{T, R\}) = T$ but $C(\{T, R, K\}) = R$.

What WARP means (2): fixed preferences

More generally, WARP implies that **preferences do not change**.

Following the economist **Robbins** (1932) and the sociologist **Parsons** (1932), many economists build on this idea to defend a strict **separation between economics, sociology and psychology**.

Milton Friedman, *Price Theory* (1962)

"Economic theory takes wants as fixed. This is primarily a case of division of labor. The economist has little to say about the formation of wants; this is the province of the psychologist. The economist's task is to trace the consequences of any given set of wants. The legitimacy of and justification for this abstraction must rest ultimately, as with any other abstraction, on the light that is shed and the power to predict that is yielded by the abstraction."

5

Cardinality and ordinality

A ranking or a quantity? What an ordinal measure tells us, what a cardinal one adds, and which theories need the extra.

An intuitive approach

Measuring temperature

- Not unique: Celsius and Fahrenheit.
- But differences are meaningful: going from 9 to 10 ℃ or from 100 to 101 ℃ is the same rise, 1 ℃ = 1.8 ℉.

Measuring distance

- Not unique: kilometres and miles.
- But going from 9 to 10 km or from 100 to 101 km is the same step, 1 km ≈ 0.62 miles.

Measuring the best film

Two French film ratings: *Télérama* (T, TT, TTT) and *AlloCiné* ($\star$, $\star\star$, $\star\star\star$).

- Not unique either (assuming they rank films the same way).
- But it does not necessarily make sense to say

$$\text{TTT} - \text{TT} = \star\star\star - \star\star = \text{TT} - \text{T}.$$

Ordinal utilities are like film ratings: we can only say that TTT and $\star\star\star$ mean **the same rank** in terms of preference.

Ordinal measure vs cardinal measure

1. Ordinal measure

A measure that only **ranks** entities according to the degree of the property being measured:

(*) if a is greater than b, which is greater than c, then the measure of a must be greater than the measures of b and c, and the measure of b greater than that of c.

Ordinal measure vs cardinal measure

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2. Cardinal measure

A measure that is ordinal *and* also ranks the **differences** between entities according to the degree of the property:

(*) holds;

(**) if the gap between a and b is greater than the gap between c and d, then the measure of a minus that of b must be greater than the measure of c minus that of d.

Ordinal utility vs cardinal utility

Beyond ranking, cardinal utility assumes that it makes sense to **compare differences** between measurements. For a utility function: the gaps between the utilities of two goods have a meaning, and these gaps can be compared.

Drawback

By taking seriously the measurement of utility and the meaning of utility gaps, it adds to ordinal utility a **contestable and unobservable** psychological hypothesis. Ordinal utility, by contrast, only assumes a ranking that matches the agent's observed choices.

Advantage

Cardinal utility offers a **richer interpretation** and lets us talk about more things (e.g. expected utility, diminishing marginal utility).

Telling them apart mathematically

How can we distinguish an ordinal measure from a cardinal one **mathematically**?

More information

A cardinal utility measure contains **more information**, since it tells us about

- not only the ranking of goods or bundles of goods;
- but also, for instance, the differences in intensity.

Hence an ordinal utility survives *any* increasing transformation φ (theorem above), while a cardinal one survives only those that preserve the ranking of differences, $\varphi(u) = \alpha u + \beta$ with $\alpha > 0$.

Properties of preferences that presuppose a cardinal utility

The independence axiom

$\succsim$ satisfies independence if, whenever $x \succ y$, then for all $\alpha \in (0,1)$ and all z:
$\alpha x + (1-\alpha)z \succ \alpha y + (1-\alpha)z$.

One can then show (with a continuity condition) that $\succsim$ is represented by a u such that

$$u(\alpha x + (1-\alpha)y) = \alpha u(x) + (1-\alpha)u(y),$$

and such a u is unique up to $a\,u + b$ with $a > 0$: a **cardinal** utility (expected utility).

Convexity of preferences

$\succsim$ is convex if, whenever $x \succsim y$, then for all $\alpha \in (0,1)$: $\alpha x + (1-\alpha)y \succsim y$.

This corresponds to a weak form of concavity of the utility function (**quasi-concavity**). Note: quasi-concavity survives every increasing transformation, so this property is in fact compatible with a purely ordinal utility.

Exercise: ordinal or cardinal?

	u_1	u_2	u_3	u_4	u_5
beer	4	10	20	4	7
donut	3	9	18	2	2
pretzel	2	8	16	1	3
herbal tea	1	7	14	0	9

Question

Which groups of the utilities on the left

- can constitute an **ordinal** measure (of the same preference)?
- can constitute a **cardinal** measure?

6

Critiques of rationality and experimental economics

Do real people choose the way the axioms say? Intransitive choices, endowment, attraction and reactance effects, and bounded rationality.

Falsifying transitivity

- We have seen that **transitivity** is a **necessary condition** for an ordinal utility to represent the agent's choices.
- Countless experiments show that individuals **do not always** satisfy transitivity.
- We will look at one that borrows ideas from **social choice theory** and leads to the hypothesis of **multiple selves** within the individual.

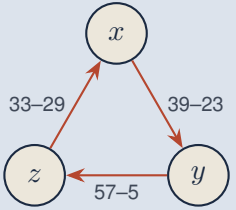
May (1954): choosing a marriage partner

May asks his students to rank three "potential marriage partners" x, y, z, described along three criteria:

	intelligence	looks	wealth
x	very intelligent	plain-looking	fairly well off
y	stupid	good-looking	poor
z	fairly intelligent	fairly good-looking	rich

Then he asks them to compare the partners **two by two**.

Results (62 students)



$x \succ y,\ y \succ z,\ z \succ x$: a **cycle**.

May (1954): an internal vote

May (1954)

"What is the significance of this experiment? Of course it does not prove that individual patterns are always intransitive. It does, however, suggest that where choice depends on conflicting criteria, preference patterns may be intransitive unless one criterion dominates."

Later in the paper, May explains this situation by an **analogy with social choice theory**:

- the candidates in the election are the marriage partners;
- the criteria are the voters.

The individual's decision would then fall under **Arrow's impossibility theorem**:

- an internal deliberation between the criteria, whose outcome is given by a voting procedure;
- such a procedure (here, pairwise majority) respects the independence of irrelevant alternatives and no criterion dictates, so, by Arrow's theorem, it cannot guarantee transitivity.

The endowment effect

The **endowment effect**, documented by Kahneman, Knetsch and Thaler (1990), is the result of a classic experiment showing that people attach **more value to what they own**.

The experiment (basic version; there are several variants)

- Two objects: for example, a mug and a pair of scissors.
- Subjects are split into two groups: everyone in one group is given a mug, everyone in the other group a pair of scissors.
- Each subject is then asked whether they want to swap their gift for the other object.

Result: in *both* groups, well over 50% refuse to swap — whereas, if gifts were randomly assigned and tastes stable, about half should want the other object.

The endowment effect: explanations

- **Loss aversion and/or a reference point** à la Kahneman and Tversky: we will come back to this in the next part.
- **Psychological inertia** (David Gal) or the **status quo bias** (Samuelson and Zeckhauser): the agent generally prefers not to change anything.
- **Psychological theories of ownership:** the literature is large, but some of these theories suggest that ownership gives the owned object a value that cannot be transferred to another object, which comes from the subject's identification — of their "self" — with the object.

Remark

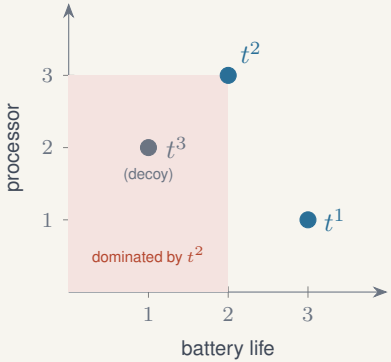
The endowment effect can be reduced by having subjects play **trading games**. It is also smaller among **professional traders**.

The attraction (decoy) effect

A bias showing the effect of a **reference** on choice. Goods have different attributes — for a phone, its **battery life** and the **power of its processor**. This induces **violations of WARP**. Consider three phones (battery, processor):

$$t^1 = (3, 1), \quad t^2 = (2, 3), \quad t^3 = (1, 2).$$

- Suppose the agent is indifferent between t^1 and t^2 when only these two phones are offered.
- But when t^1, t^2 and t^3 are offered, she chooses t^2, because t^3 is **worse than** t^2 on both attributes.



Reactance

Mazis, Settle and Leslie (1973)

- In 1972, Miami-Dade County (Florida) **banned phosphate detergents**.
- In reaction, some people started smuggling phosphate detergent and/or stockpiling it for 20 years.

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We observe: $C(\{x, y, z\}) = z$ and $C(\{y, z\}) = y$.

Bounded rationality

We owe to **Herbert Simon** the notion of **bounded rationality**: agents have some capacity to be rational, but this capacity is limited.

Satisficing (Simon)

Among the possible options, the agent does not necessarily choose the best one: she chooses the **first satisfactory** option she comes across. This is called **procedural rationality**.

The psychologist **Daniel Kahneman**, for his part, argues that agents use two systems to make decisions:

System 1

Fast, cognitively cheap, relies on heuristics; efficient most of the time.

System 2

Slow, reflective, costly, but much more accurate.

Take-aways

1. The marginalists' utility was attacked as **unmeasurable**; their answers (comparison, prices, money) pointed towards what can be compared and observed.
2. Behaviourism, logical empiricism and Popper pushed economics towards **observable, falsifiable** notions — while keeping a folk psychology of beliefs and desires.
3. With Pareto, **preference** comes first: a utility exists if and only if preferences are **complete and transitive**, and it is unique only up to an increasing transformation.
4. Samuelson's **WARP** restates this in terms of observed **choices**, and makes utility maximisation falsifiable — at the price of fixed, context-free preferences.
5. **Ordinal** utility only ranks; **cardinal** utility also ranks differences: richer, but a stronger, unobservable psychological hypothesis.
6. Experiments (May's cycles, endowment, attraction and reactance effects) show choices that **violate** these axioms; bounded rationality and the two systems offer psychological explanations.