INTRODUCTION TO ECONOMICS FOR PSYCHOLOGISTS: HISTORY, CONCEPTS, AND PSYCHOLOGICAL FOUNDATIONS

PART 3

What Ought to Be: Normative Economics and Collective Choice from Borda to Arrow

Niels Boissonnet

Université Paris Cité · Second-year Psychology · 2022/23

Outline

Positive and normative economics
The social welfare function
Pareto optimality and the welfare theorems
Social choice theory
Which axiom should we give up?

Today's question: what **ought** a society to choose — and can individual preferences be turned into a collective choice at all?

1

Positive and normative economics

What is versus what ought to be.

Three kinds of knowledge

"A positive science may be defined as a body of systematized knowledge concerning what is; a normative or regulative science as a body of systematized knowledge discussing criteria of what ought to be, and concerned therefore with the ideal as distinguished from the actual; an art as a system of rules for the attainment of a given end. The object of a positive science is the establishment of uniformities, of a normative science the determination of ideals, of an art the formulation of precepts."

John Neville Keynes, *The Scope and Method of Political Economy*, 1890

Positive economics and normative economics

Positive economics: what *is*

Example: what are the consequences of a given tax on relative prices?

Normative economics: what *ought to be*

Example: is a progressive tax fairer?

Is transitivity a normative or a positive principle?

Positive economics and normative economics

Positive economics: what *is*

Example: what are the consequences of a given tax on relative prices?

Normative economics: what *ought to be*

Example: is a progressive tax fairer?

Is transitivity a normative or a positive principle?

- We saw that people violate it: as a **positive** description, it fails.
- Yet we can regard it as **normatively** well founded: someone with intransitive preferences can be turned into a **money pump**.

Positive economics and normative economics

Positive economics: what *is*

Example: what are the consequences of a given tax on relative prices?

Normative economics: what *ought to be*

Example: is a progressive tax fairer?

Is transitivity a normative or a positive principle?

- We saw that people violate it: as a **positive** description, it fails.
- Yet we can regard it as **normatively** well founded: someone with intransitive preferences can be turned into a **money pump**.

The same principle can fail as a description of behaviour and still hold as a standard of good reasoning.

2

The social welfare function

How should a society rank the ways its goods could be shared?

The social welfare function

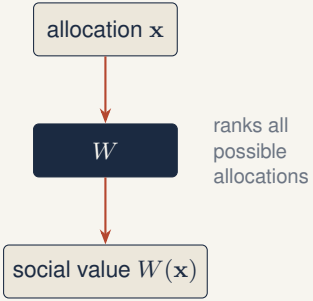
- x_i: the quantity of goods individual i receives;
- each individual i has a utility function u_i;
- an **allocation** assigns a quantity of each good to each individual:

$$\mathbf{x} = (x_1, \ldots, x_n).$$

Definition

A **social welfare function** W assigns a value $W(\mathbf{x})$ to each allocation $\mathbf{x}$.

Goal: find the allocation that maximizes $W(\mathbf{x})$.



Example: taxation

- Each individual i has an income r_i.
- The State chooses a tax rate t_i and a social benefit s_i for each individual, so as to maximize

$$W(r_1 - r_1 t_1 + s_1,\ \ldots,\ r_n - r_n t_n + s_n).$$

The allocation

Here an allocation is the vector of incomes after taxes and benefits:

$$(x_1, \ldots, x_n) = (r_1 - r_1 t_1 + s_1,\ \ldots,\ r_n - r_n t_n + s_n).$$

Defining social welfare: holism versus individualism

Holist view

The State is a reality in itself, superior to the individuals who make up the social group and entirely distinct from them.

- Social welfare is what the State says it is, i.e. it depends on the State's utility u_E:

$$W(\mathbf{x}) = W(u_E(\mathbf{x})).$$

Individualist view

The State has no reality of its own. It is only a tool that individuals use to pursue their own interests by means other than the market.

- **Consumer sovereignty**;
- social welfare depends on the individual utility functions:

$$W(\mathbf{x}) = W(u_1(\mathbf{x}), \ldots, u_n(\mathbf{x})).$$

Utilitarianism

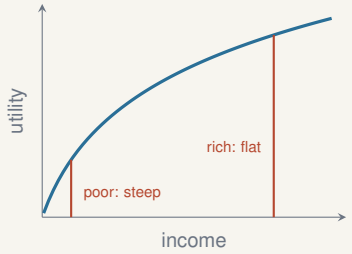
Utilitarianism builds in a form of **impartiality**: every individual's utility counts in the same way.

Utilitarian social welfare function

$$W(\mathbf{x}) = u_1(\mathbf{x}) + \cdots + u_n(\mathbf{x}).$$

Taxation

If marginal utilities are decreasing, the rich are taxed more and the poor receive benefits.



A euro taken from the rich costs less utility than it brings to the poor.

Problem

This assumes that individuals' utilities are **cardinal** and **comparable** across people.

The Rawlsian social welfare function

In his famous book *A Theory of Justice* (1971), John Rawls identifies two principles of a just society, ordered **lexicographically** (the first always takes priority):

1. The liberty principle

Each person has an equal right to the most extensive total system of equal basic liberties compatible with a similar system of liberty for all.

2. The difference principle

Social and economic inequalities must be arranged so that they are (a) to the greatest benefit of the least advantaged, and (b) attached to offices and positions open to all under conditions of fair equality of opportunity.

The difference principle calls for maximizing the utility of the **worst-off** person. We seek to maximize

$$W(\mathbf{x}) = \min_i u_i(\mathbf{x})$$

3

Pareto optimality and the welfare theorems

Efficiency, and what markets have to do with it.

Pareto optimality

Intuition: an allocation is Pareto optimal if no other allocation makes someone better off without making anyone worse off.

Definition

An allocation $\mathbf{x}$ **Pareto dominates** $\mathbf{x}'$ if (1) for every i, $u_i(x_i) \geq u_i(x'_i)$, and (2) there is some j with $u_j(x_j) > u_j(x'_j)$.
An allocation $\mathbf{x}$ is a **Pareto optimum** if no allocation Pareto dominates it.

Remark: Pareto optimality can be seen as the criterion of **efficiency**.

Example: which allocations are Pareto optimal?

	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$
u_1	1	2	4	3
u_2	1	1	2	2
u_3	1	2	1	2

Pareto optimality

Intuition: an allocation is Pareto optimal if no other allocation makes someone better off without making anyone worse off.

Definition

An allocation $\mathbf{x}$ **Pareto dominates** $\mathbf{x}'$ if (1) for every i, $u_i(x_i) \geq u_i(x'_i)$, and (2) there is some j with $u_j(x_j) > u_j(x'_j)$.
An allocation $\mathbf{x}$ is a **Pareto optimum** if no allocation Pareto dominates it.

Remark: Pareto optimality can be seen as the criterion of **efficiency**.

Example: which allocations are Pareto optimal?

Answer: $\mathbf{x}_3$ and $\mathbf{x}_4$. Both $\mathbf{x}_1$ and $\mathbf{x}_2$ are dominated by $\mathbf{x}_4$; neither $\mathbf{x}_3$ nor $\mathbf{x}_4$ dominates the other.

	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$
u_1	1	2	**4**	**3**
u_2	1	1	**2**	**2**
u_3	1	2	**1**	**2**

The fundamental theorems of welfare economics

In an exchange economy like the one we studied in Part 1 of the course, each price induces an allocation.

- What is the link between **equilibrium prices** and **Pareto optima**?

The welfare theorems (under certain assumptions)

1. An equilibrium price induces an allocation that is **Pareto optimal**.
2. Every Pareto-optimal allocation can be induced by an equilibrium price, after a suitable redistribution of the initial endowments.

This suggests that the argument "the market is efficient" is correct. Of course, it rests on very restrictive assumptions (**perfect competition**).

4

Social choice theory

From individual preferences to a collective choice: Borda, majority, and Arrow's impossibility theorem.

The social choice function

The welfare functions we have seen rely on **cardinal and comparable** utilities. Can we build a similar concept with **ordinal** utilities, i.e. with preferences only?

Definition

A **social choice function** C gives, for each profile of preferences $(\succsim_1, \ldots, \succsim_n)$ (all transitive and complete), a chosen allocation

$$\mathbf{x} = C(\succsim_1, \ldots, \succsim_n).$$

Can we imagine a social choice function that is represented by a utilitarian social welfare function?

- Could we not use, for each individual i, a utility function u_i that represents $\succsim_i$?
- If not, what is the problem?

Social choice functions and voting rules

A social choice function can be seen as giving the outcome of a **vote** under a particular voting rule. Write $\succsim = (\succsim_1, \ldots, \succsim_n)$ for the preference profile.

Plurality rule ("simple majority")

Let $n^{\mathbf{x}}(\succsim)$ be the number of individuals who rank $\mathbf{x}$ first. If $\mathbf{x} = C^M(\succsim)$, then

$$n^{\mathbf{x}}(\succsim) \geq n^{\mathbf{y}}(\succsim) \quad \text{for every } \mathbf{y}.$$

Borda rule

Let $n_i^{\mathbf{x}}$ be the rank of $\mathbf{x}$ in i's preference (1 = best). If $\mathbf{x} = C^B(\succsim)$, then

$$\textstyle\sum_i n_i^{\mathbf{x}} \leq \sum_i n_i^{\mathbf{y}} \quad \text{for every } \mathbf{y}.$$

Arrow's four axioms

Universal domain

Every profile of preferences is possible.

Pareto efficiency

If $C(\succsim) = \mathbf{x}$, then $\mathbf{x}$ is a Pareto optimum.

Independence of irrelevant alternatives

Let $\succsim$ and $\succsim'$ be two profiles such that, for every i, $\mathbf{x} \succsim_i \mathbf{y} \iff \mathbf{x} \succsim'_i \mathbf{y}$. If $\mathbf{x} = C(\succsim)$, then $\mathbf{y} \neq C(\succsim')$.

Non-dictatorship

No individual's favourite option is chosen whatever his or her own and the others' preferences: there is no i such that, for every profile $\succsim$,

$$C(\succsim) = \mathbf{x} \iff \mathbf{x} \succsim_i \mathbf{y} \text{ for every } \mathbf{y}.$$

Arrow's impossibility theorem

Theorem (Arrow, 1951)

No social choice function satisfies universal domain, Pareto efficiency, independence of irrelevant alternatives and non-dictatorship.

Conclusion

If you are against dictatorship, the voting rule you choose will violate universal domain, Pareto efficiency or independence of irrelevant alternatives.

Why does it matter?

Because these assumptions are *desirable*.

Arrow's impossibility theorem

Theorem (Arrow, 1951)

No social choice function satisfies universal domain, Pareto efficiency, independence of irrelevant alternatives and non-dictatorship.

Conclusion

If you are against dictatorship, the voting rule you choose will violate universal domain, Pareto efficiency or independence of irrelevant alternatives.

Why does it matter?

Because these assumptions are *desirable*. Which one could we do without?

5

Which axiom should we give up?

Every escape route gives something up. Which axiom do the usual rules violate, and which can we drop?

The plurality rule violates independence of irrelevant alternatives

Three candidates: MAC, MEL, LEP

3 voters	2 voters	2 voters
MAC	LEP	MEL
MEL	MEL	LEP
LEP	MAC	MAC

First places: MAC 3, LEP 2, MEL 2

$\Longrightarrow$ **MAC wins.**

Two candidates: MAC, MEL (LEP withdraws)

3 voters	2 voters	2 voters
MAC	**MEL**	**MEL**
MEL	MAC	MAC

First places: MAC 3, MEL 4

$\Longrightarrow$ **MEL wins.**

No voter changed his or her mind about MAC versus MEL. Yet removing LEP, an "irrelevant" alternative, reverses the collective choice between them.

MAC, MEL, LEP: Macron, Mélenchon, Le Pen (French presidential election, 2022).

The utilitarian rule

$$C(\succsim) = \arg\max_{\mathbf{x}} W(\mathbf{x}) = \arg\max_{\mathbf{x}} \; u_1(\mathbf{x}) + \cdots + u_n(\mathbf{x}). \tag{1}$$

Problem

There is not one but **many** utility functions that represent each voter's preference $\succsim_i$. So for a given profile $\succsim$, there are many W, which recommend different things: (1) is not well defined.

Example: two alternatives $\mathbf{x}, \mathbf{y}$; two voters: a prefers $\mathbf{x}$, b prefers $\mathbf{y}$.

One choice of utilities

	$\mathbf{x}$	$\mathbf{y}$
u_a	1	0
u_b	0	1000
W	1	**1000**

C depends mostly on b's preference: $\mathbf{y}$ is chosen.

The opposite choice

	$\mathbf{x}$	$\mathbf{y}$
u_a	1000	0
u_b	0	1
W	**1000**	1

C depends mostly on a's preference: $\mathbf{x}$ is chosen.

The Borda rule

The Borda rule is a special case of the utilitarian rule, in which

$$u_i(\mathbf{x}) = \text{number of alternatives} - \text{rank of } \mathbf{x} \text{ in } \succsim_i,$$

$$C(\succsim) = \arg\max_{\mathbf{x}} W(\mathbf{x}) = \arg\max_{\mathbf{x}} \; u_1(\mathbf{x}) + \cdots + u_n(\mathbf{x}).$$

Example: four alternatives

rank in $\succsim_i$	1st	2nd	3rd	4th
u_i (Borda points)	3	2	1	0

Exercise

Show that the Borda rule violates independence of irrelevant alternatives.

Pareto efficiency: should we keep it?

Pareto efficiency

The voting rule must not select a candidate if all citizens unanimously judge it worse than another one.

Desirable? Yes. It simply says that if everyone agrees that one thing is worse than another, it should not be chosen.

Independence of irrelevant alternatives: should we keep it?

Independence of irrelevant alternatives

If the voting rule ranks candidate A above candidate B when there are only these two candidates, it must also rank A above B when the options are A, B and C (given that individual preferences have not changed).

Main criticism: it rules out **every kind of interpersonal comparison of utility**, whereas some of them may be legitimate — for instance when the social choice function is interpreted as a welfare function.

"A rabid ordinalist might object that it is unscientific to include in our models things such as feelings or experience, because they are not observable or subject to investigation by market experimentation. This view may serve us well in the positive theory of demand, where revealed preference analysis says all there is to say. But when one speaks of welfare economics and social choice, human perceptions and feelings are of the essence: without them, it is not clear what is being discussed. To trim experience from a model of social welfare in the name of Occam's Razor is to kill the subject by cutting out its heart."

— D. Pearce, 1995

Universal domain: should we keep it?

Universal domain

Individual preferences can be distributed in society in any possible way.

Criticism: societies have cultural norms that prevent just any distribution of preferences from developing.

Example: the left–right axis in politics restricts the domain



- If my favourite candidate is Hamon, I usually prefer Macron to Le Pen; if my favourite is Macron, I prefer Hamon to Mélenchon.
- Under this restriction, there is **no impossibility** (Black, 1948): under majority voting, the candidate elected is the favourite of the **median voter**.

Take-aways

1. **Positive** economics describes what is; **normative** economics asks what ought to be. Transitivity fails as a description but stands as a norm.
2. A **social welfare function** ranks allocations. Utilitarianism adds utilities; Rawls maximizes the utility of the worst-off. Both compare utilities across people.
3. **Pareto optimality** captures efficiency; under perfect competition, market equilibria are Pareto optimal.
4. **Arrow's theorem**: with ordinal preferences only, no social choice function satisfies universal domain, Pareto efficiency, independence of irrelevant alternatives and non-dictatorship.
5. Escape routes: give up **independence** (allow interpersonal comparisons, as Borda and utilitarianism do) or give up **universal domain** (a left–right axis, the median voter).