INTRODUCTION TO ECONOMICS FOR PSYCHOLOGISTS: HISTORY, CONCEPTS, AND PSYCHOLOGICAL FOUNDATIONS

PART 5

Strategic Minds: Game Theory from Nash to the Laboratory

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So far…

We have seen how economists analyse the behaviour of **isolated individuals**

- whose payoffs do *not* depend on what other people do.

Question: what happens when the consequences of my choice **depend on the choices of others**?

Answer: **game theory**!

Outline

Non-cooperative collective rationality
Nash equilibrium in pure strategies
Nash equilibrium and Pareto optimum
Nash equilibrium in mixed strategies
Experiments on cooperation games
Coordination games
Games and time
Incentives and asymmetric information

Payoff matrices: the **row player**'s payoff comes first (red), the **column player**'s second (blue).

1

Non-cooperative collective rationality

Rational players who know that the others are rational too.

The prisoner's dilemma (Tucker)

Two accomplices in a crime, held in **separate cells**, are each asked whether they want to denounce their partner:

- if only one denounces, he goes free and the other gets the **maximum sentence** (10 years);
- if both denounce, they both get a **lighter sentence** (5 years);
- if neither denounces, the sentence is **minimal** (6 months), for lack of evidence.

	Stay silent	**Denounce**
Stay silent	$-0.5, -0.5$	$-10, 0$
Denounce	$0, -10$	$-5, -5$

Payoffs = minus the years in prison.

Interactive rationality

In game theory, we assume that players

- . . . maximise their payoffs;
- . . . assume that the others maximise their payoffs too.

So players are not only rational: they also **assume that the others are rational**.

Common belief in rationality

Each agent knows that the others are rational, but also knows that the others know that he is rational, and knows that the others know that he knows that they are rational, etc.

- Slightly **less restrictive** (but less well known) than Nash equilibrium.

Common belief in rationality at work

Common belief in rationality

Each agent knows that the others are rational, but also knows that the others know that he is rational, and knows that the others know that he knows that they are rational, etc.

	A1	A2	A3
B1	$2,1$	$3,0$	$0,0$
B2	$1,2$	$2,5$	$-1,5$
B3	$1,2$	$1,1$	$1,2$

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$\Longrightarrow$

	A1	A3
B1	$2,1$	$0,0$
B3	$1,2$	$1,2$

B2 always does worse than B1, so a rational row player never plays it. Knowing this, the column player sees that A2 always does worse than A1, so she never plays it either.

2

Nash equilibrium in pure strategies

No one wants to change strategy, given what the others do.

Nash equilibrium

A **Nash equilibrium** is a set of strategies chosen by the players such that **none of them has an interest in changing strategy**, given what the other players play at this equilibrium.

Suppose there are only two players (this extends easily to n players):

- s_1 the strategy of player 1 and s_2 that of player 2;
- $s = (s_1, s_2)$ a **strategy profile**;
- $u_1(s_1, s_2)$ and $u_2(s_1, s_2)$ the utilities of players 1 and 2 if they choose s_1 and s_2.

Definition

(s_1, s_2) is a Nash equilibrium if, for every other strategy s,

$$u_1(s_1, s_2) \geq u_1(s, s_2) \qquad \text{(player 1 does not want to deviate)}$$

$$u_2(s_1, s_2) \geq u_2(s_1, s) \qquad \text{(player 2 does not want to deviate)}$$

Finding the Nash equilibria

	Accept	Fight
Challenge	$2, 1$	$0, 0$
Do nothing	$1, 2$	$1, 2$

- (Do nothing, Accept)?

Finding the Nash equilibria

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- (Challenge, Fight)? **No**: both players deviate.
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Finding the Nash equilibria

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- (Do nothing, Accept)? **No**: the challenger deviates (to Challenge).
- (Challenge, Accept)? **Yes**.
- (Challenge, Fight)? **No**: both players deviate.
- (Do nothing, Fight)? **Yes**.

Two Nash equilibria: (Challenge, Accept) and (Do nothing, Fight).

Common belief in rationality vs Nash equilibrium

Question: how are Nash equilibrium and common belief in rationality related?

Nash equilibrium = common belief in rationality + simple beliefs

Simple beliefs

Players presume that the belief they attribute to their opponent about their own belief is always **correct**.

- Yet a player could think that the other player attributes to him a belief (about his own belief) that is **false** but **rational** (in the sense of common belief in rationality).

Advantage of Nash equilibrium

It reduces indeterminacy: fewer Nash equilibria, hence less indeterminacy.

But

Nash equilibrium is less general and rests on questionable assumptions.

3

Nash equilibrium and Pareto optimum

When individual rationality leads to a collectively bad outcome.

The prisoner's dilemma, again

A Nash equilibrium is a situation in which each player chooses a strategy and **no player has an interest in deviating**, i.e. in choosing another strategy.

	Stay silent	**Denounce**
Stay silent	$-0.5, -0.5$	$-10, \underline{0}$
Denounce	$\underline{0}, -10$	**-5**, **-5**

Underlined: best response to the other's strategy.

Shaded: Nash equilibrium.

Why is this surprising?

- Both players are **better off** if neither denounces; yet if they are rational and believe the other is rational, they will **denounce**.
- Rationality does **not always lead to a Pareto optimum** (contrary to the idea behind the theorems of welfare economics).

Applications of the prisoner's dilemma (1)

Nuclear proliferation

Instead of two prisoners, take two countries wondering whether to develop nuclear weapons.

	No nuclear weapon	**Nuclear weapon**
No nuclear weapon	$-0.5, -0.5$	$-10, 0$
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The tragedy of the commons (G. Hardin, 1968)

Two individuals exploit a common resource. Each has an interest in consuming more than they should if the resource is not to run dry.

Applications of the prisoner's dilemma (2)

The market for information

Competition between media pushes them to favour news that is fast (first on the scene), poorly substantiated (well-known but not very competent pundits) or uninteresting (buzz) over in-depth, quality news $\Longrightarrow$ errors are **shared by all**.

Applications of the prisoner's dilemma (2)

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Advertising and marketing

Firms invest in advertising and marketing (9.5% on average) rather than innovate or raise wages; but if they do not, they die.

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Group work

Everyone has an interest in relying on the work of the others.

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Group work

Everyone has an interest in relying on the work of the others.

Cycling races

Nobody has an interest in riding at the front of the peloton.

The public goods game

Same idea as the prisoner's dilemma, except that there are **as many players as we like**, not just 2.

The game

- Several players (say 4);
- each receives 10 €;
- each player i decides to put an amount r_i of these 10 € into the "public good";
- at the end, player i earns

$$10 - r_i + 0.5 \times (r_1 + r_2 + r_3 + r_4).$$

In other words, all players would be **better off** if everyone contributed to the public good, but everyone also has an interest in **not contributing** and benefiting from the others' contributions.

Conclusion: the Nash equilibrium is that nobody invests in the public good, whereas the Pareto optimum is to invest.

Applications of the public goods game

In some domains, the public goods game justifies **state intervention** or the existence of public services, for instance for **public goods**, which are:

- **non-rival**: one individual's consumption of the good does not prevent others from consuming it;
- **non-excludable**: it is impossible to prevent people from consuming it.

Examples

- The army, the police (cf. Nozick's libertarianism);
- a lighthouse;
- schools and hospitals (less clear-cut, since exclusion is possible). These are called **merit goods**.

4

Nash equilibrium in mixed strategies

Choosing a probability over actions rather than an action.

Existence of a Nash equilibrium

A Nash equilibrium is a set of strategies chosen by the players such that no player has an interest in deviating, i.e. in choosing another strategy, given what the other players play.

	A1	**A2**
B1	$4, 4$	$2, 3$
B2	$5, 1$	$0, 2$

- Here there is **no Nash equilibrium in pure strategies**...
- ... but there is a Nash equilibrium in **mixed strategies**.

Mixed strategies

Mixed strategy

A strategy in which the agent chooses not an action, but a **probability of taking each action**.

The agents have utilities u_1 and u_2.

- s_{11} and s_{12}: the pure strategies of agent 1;
- s_{21} and s_{22}: the pure strategies of agent 2;
- $p(s_{1i})$: the probability that player 1 plays s_{1i};
- $q(s_{2i})$: the probability that player 2 plays s_{2i}.

Nash equilibrium in mixed strategies

Define the **expected utilities**

$$\begin{aligned} U_1(p,q) &= p(s_{11})q(s_{21})\,u_1(s_{11},s_{21}) + p(s_{12})q(s_{21})\,u_1(s_{12},s_{21}) \\ &\quad + p(s_{11})q(s_{22})\,u_1(s_{11},s_{22}) + p(s_{12})q(s_{22})\,u_1(s_{12},s_{22}) \\ U_2(p,q) &= p(s_{11})q(s_{21})\,u_2(s_{11},s_{21}) + p(s_{12})q(s_{21})\,u_2(s_{12},s_{21}) \\ &\quad + p(s_{11})q(s_{22})\,u_2(s_{11},s_{22}) + p(s_{12})q(s_{22})\,u_2(s_{12},s_{22}) \end{aligned}$$

Definition

(p,q) is a Nash equilibrium if, for all other strategies $\hat{p},\hat{q}$,

$U_1(p,q) \geq U_1(\hat{p},q)$ (player 1 does not want to deviate)

$U_2(p,q) \geq U_2(p,\hat{q})$ (player 2 does not want to deviate)

Example: computing the mixed strategies

	A1	A2
B1	$4, 4$	$2, 3$
B2	$5, 1$	$0, 2$

Let p be the strategy of player 1 (the probability that he plays B1) and q the strategy of player 2 (the probability that she plays A1). Each chooses p (player 1) or q (player 2) so as to **maximise their expected utility**.

Example: computing the mixed strategies

	A1	A2
B1	$4, 4$	$2, 3$
B2	$5, 1$	$0, 2$

Let p be the strategy of player 1 (the probability that he plays B1) and q the strategy of player 2 (the probability that she plays A1). Each chooses p (player 1) or q (player 2) so as to **maximise their expected utility**.

Expected utilities

$$U_1 = p\,(4q + 2(1-q)) + (1-p)(5q + 0\,(1-q)) \qquad = p\,(2 - 3q) + 5q$$
$$U_2 = q\,(4p + 1\,(1-p)) + (1-q)(3p + 2(1-p)) \qquad = q\,(2p - 1) + p + 2$$

Example: best responses and equilibrium

p maximises $p(2-3q)+5q$ and q maximises $q(2p-1)+p+2$. So the best strategies of the two players are

$$p^* = \begin{cases} 0 & \text{if } q > 2/3 \\ \text{anything} & \text{if } q = 2/3 \\ 1 & \text{if } q < 2/3 \end{cases} \qquad q^* = \begin{cases} 0 & \text{if } p < 1/2 \\ \text{anything} & \text{if } p = 1/2 \\ 1 & \text{if } p > 1/2 \end{cases}$$

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- If player 1 chooses $p > 1/2$, player 2 should choose $q = 1 > 2/3$, and then player 1 would deviate to $p = 0 < 1/2$: **not a Nash equilibrium**.
- Same reasoning if $p < 1/2$, $q > 2/3$ or $q < 2/3$.

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- Same reasoning if $p < 1/2$, $q > 2/3$ or $q < 2/3$.

The Nash equilibrium is therefore $(p = 1/2,\ q = 2/3)$: each player makes the other **indifferent** between their two actions.

Example: rock, paper, scissors

- Two players (1 and 2) who can win -1, 0 or 1;
- each has three strategies: rock R, paper P, scissors S;
- $\mu_i(R)$, $\mu_i(P)$ and $\mu_i(S)$: the probabilities that player i plays these strategies.

Claim

The unique Nash equilibrium is

$$(\mu_1(R), \mu_1(P), \mu_1(S)) = (1/3, 1/3, 1/3) = (\mu_2(R), \mu_2(P), \mu_2(S)).$$

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- $\mu_i(R)$, $\mu_i(P)$ and $\mu_i(S)$: the probabilities that player i plays these strategies.

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The unique Nash equilibrium is

$$(\mu_1(R), \mu_1(P), \mu_1(S)) = (1/3, 1/3, 1/3) = (\mu_2(R), \mu_2(P), \mu_2(S)).$$

Step 1: this is a Nash equilibrium. Suppose $\mu_i(R) = \mu_i(P) = \mu_i(S) = 1/3$. Then player 2 has no interest in changing strategy: whatever she plays, her expected utility is

$$\mu_2(R) \cdot (1/3 - 1/3) + \mu_2(P) \cdot (1/3 - 1/3) + \mu_2(S) \cdot (1/3 - 1/3) = 0.$$

The same holds for player 1, so this is a Nash equilibrium.

Rock, paper, scissors: there is no other equilibrium

By contradiction: suppose there is an equilibrium profile $(\alpha_1, \alpha_2) \neq (\mu_1, \mu_2)$.

Step 2. First, in (α_1, α_2) no player can play a strategy with probability 1 (why?). It follows that no player can play a strategy with probability 0. Suppose, say, $\alpha_1(S) = 0$: player 1 mixes only R and P, so against him P does strictly better than R and player 2 never plays R; but then, for player 1, S does strictly better than P, so he plays R with probability 1. A contradiction.

Step 3. We can therefore assume that

$$\alpha_2(R) = \max\big(\alpha_2(R), \alpha_2(P), \alpha_2(S)\big) > 1/3 > \min\big(\alpha_2(R), \alpha_2(P), \alpha_2(S)\big) = \alpha_2(P).$$

The expected utility of player 1 is

$$\alpha_1(R) \cdot \underbrace{\big(\alpha_2(S) - \alpha_2(P)\big)}_{\geq 0} + \alpha_1(P) \cdot \underbrace{\big(\alpha_2(R) - \alpha_2(S)\big)}_{\geq 0} + \alpha_1(S) \cdot \underbrace{\big(\alpha_2(P) - \alpha_2(R)\big)}_{< 0}$$

So player 1's best strategy is to play $\alpha_1(S) = 0$, which contradicts Step 2.

How to read a mixed strategy

Mixed strategy

A strategy in which the agent chooses not an action, but a probability of taking each action.

Three possible interpretations:

1. the player **deliberately randomises** her action;
2. the randomisation captures the other player's **guess** about her strategy;
3. the game is **repeated** many times, and the player plays each strategy with a given frequency.

Nash's key result: there always exists a Nash equilibrium in mixed strategies (in any game with finitely many players and strategies).

5

Experiments on cooperation games

What do real people do in the prisoner's dilemma and its cousins?

Experimental results

Prisoner's dilemma

- Roughly **one third** of subjects choose to cooperate.
- Prisoner's dilemmas repeated several times. Several strategies:
  - cooperate, or do not cooperate, **unconditionally**, i.e. whatever the opponent chose before;
  - **tit for tat**: cooperate in the first round, then cooperate only if the other cooperated in the previous round (Axelrod).

Public goods game

- As in the prisoner's dilemma, some subjects cooperate, i.e. they put money into the public good.
- In repeated public goods games, they cooperate **less and less**, unless some members are given the possibility to **pay to punish** the others (Fehr and Gächter, 2000).

The dictator game

The game

A subject receives 10 € and is asked how much of these 10 € she wants to give to another subject.

Results (Camerer, 2003)

- Only **half** of the participants offer nothing to the other player;
- a **quarter** split the amount equally.

This contradicts the **self-interest** hypothesis.

The ultimatum game

The game

Subject A receives 10 €, which she must share with another subject. After A's decision, subject B can either take what A gave him, or refuse, in which case **both get nothing**.

Experimental results (Güth et al., 1982):

- Most A subjects give **between 40% and 50%** of the 10 €. When they give less than 20%, B rejects. **This contradicts the self-interest hypothesis.**
- When B plays against a computer, some still refuse, but only smaller amounts. So there is an effect due to **misunderstanding the game** and an effect of **moral disgust**.
- Oosterbeek (2004) shows that behaviour in the ultimatum game varies a lot **across cultures**.
  - Mongolian A subjects always offer an equal share, whereas Mongolian B subjects accept any share.

Some theoretical concepts to explain these results

Pure altruism

Individuals take the utility of others into account **unconditionally**.

Reciprocity

Individuals return the favour, both positively and negatively. It is therefore **conditional**. What role do intentions play?

Inequality aversion

Individuals always want everyone to get an **equal share**.

6

Coordination games

When what I should do depends on guessing what you will do.

Keynes's beauty contest



"It is not a case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. **We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be.** And there are some, I believe, who practise the fourth, fifth and higher degrees."

Keynes (1936, p. 156)

Coordination game and mixed strategies

Two cars arrive face to face.

	Drive on the left	**Drive on the right**
Drive on the left	$1, 1$	$-1, -1$
Drive on the right	$-1, -1$	$1, 1$

- **Two equilibria in pure strategies**: (Left, Left) and (Right, Right);
- but there is another kind of equilibrium, in **mixed strategies**: each driver drives on the left with probability 1/2 and on the right with probability 1/2.

The battle of the sexes

A couple want to go to a show. They prefer to be **together in any case**, but **disagree** about which show to see.

	Dance	**Boxing**
Dance	$3, 1$	$0, 0$
Boxing	$0, 0$	$1, 3$

Coordination games and the glue of social order

Lewis, *Convention*

The philosopher David Lewis puts coordination games at the heart of the concept of **social convention**.

- Example: driving on the right or on the left.

Schelling, *The Strategy of Conflict*

Schelling notes that in many cases agents find a way to coordinate that **the theory does not explain**.

- Example: we want to meet in Paris, with three options: Plaisance metro station, Cardinal Lemoine, or the Eiffel Tower.

This ability to coordinate comes from **focal points**, which are more **salient**. Culture of course plays an important role in this salience.

7

Games and time

Players who move one after the other.

Games and time

So far (apart from the ultimatum game), we have seen games in which players make their decisions **simultaneously**.

We can also model situations in which players **take turns**.

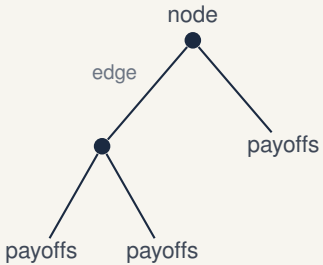
Example: the Stackelberg game

- A firm enters the market and chooses what quantity to produce.
- A second firm then enters and, given the quantity produced by the first firm, chooses in turn what quantity to produce.

Game trees

To represent these games, we use a **tree** in which:

- each **node** is a point where one (or several) players must make a decision;
- each **edge** is a possible action.

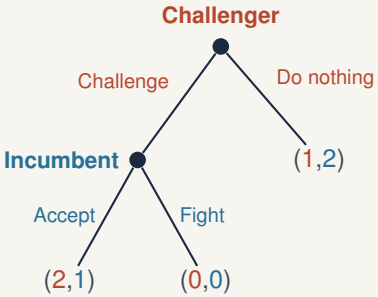


Example: entry into a market

Two players: a **challenger** firm and an **incumbent** firm.

1. First, the challenger decides whether to **enter** the competition.
2. If she does not enter, she earns 1 € and the incumbent earns 2 €.
3. If she enters, the incumbent can then
   - **accept** her entry: the incumbent earns 1 € and the challenger 2 €;
   - **fight**: both firms get 0 €.

The entry game: tree and Nash equilibria

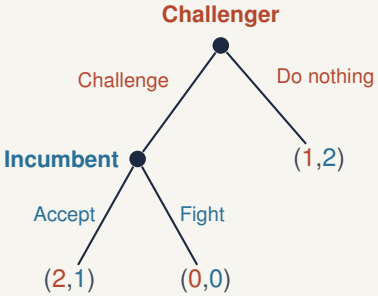


	Accept	Fight
Challenge	$2,1$	$0,0$
Do nothing	$1,2$	$1,2$

What are the Nash equilibria?

- (Do nothing, Accept)? No: the challenger deviates.
- (Challenge, Accept)? **Yes**.
- (Challenge, Fight)? No: both deviate.
- (Do nothing, Fight)? **Yes**.

The entry game: tree and Nash equilibria



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What are the Nash equilibria?

- (Do nothing, Accept)? No: the challenger deviates.
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- (Do nothing, Fight)? **Yes**.

Problem

The equilibrium (Do nothing, Fight) rests on the idea that the incumbent might choose to fight. But it has no interest in doing so: this is a **non-credible threat**.

Backward induction

Nash equilibrium is not suited to this situation, nor ever to dynamic games of this kind. We need a **new solution concept**.

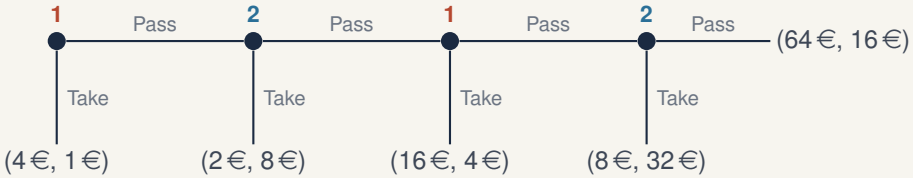
- Typical of game theory: formulate a kind of equilibrium that fits the situation well;
- also typical of game theory: find counterexamples.

Which equilibrium would work here? Backward induction

- Look at the rational decision of player n (the **last** to move); say it is action a_n.
- Require that the second-to-last player presumes that the last player will play a_n: he only considers the payoffs he can get if the next player chooses a_n. Look at the rational decision of player $n-1$, given that a_n will be chosen; say it is a_{n-1}.
- ... and so on, back to the **first** player.

In the entry game: the incumbent accepts, so the challenger enters: only (Challenge, Accept) survives.

The centipede game



Payoffs: (player 1, player 2). At each node, the player to move can *take* and stop the game, or *pass* and let the pot grow.

McKelvey and Palfrey (1992): where do games stop?

Let

- f_i be the proportion of games that stop at the i-th node;
- p_i the probability that a player stops the game at the i-th node (given that it is reached):

$$p_i = \frac{f_i}{\sum_{j=i}^{n+1} f_j}.$$

Backward induction predicts that player 1 takes at the very first node ($f_1 = 1$).

TABLE IIA

PROPORTION OF OBSERVATIONS AT EACH TERMINAL NODE

	Session		N	f_1	f_2	f_3	f_4	f_5	f_6	f_7
Four Move	1	(PCC)	100	.06	.26	.44	.20	.04		
	2	(PCC)	81	.10	.38	.40	.11	.01		
	3	(CIT)	100	.06	.43	.28	.14	.09		
	Total	1–3	281	.071	.356	.370	.153	.049		
High Payoff	4	(High-CIT)	100	.150	.370	.320	.110	.050		
Six Move	5	(CIT)	100	.02	.09	.39	.28	.20	.01	.01
	6	(PCC)	81	.00	.02	.04	.46	.35	.11	.02
	7	(PCC)	100	.00	.07	.14	.43	.23	.12	.01
	Total	5–7	281	.007	.064	.199	.384	.253	.078	.014

McKelvey and Palfrey (1992): take probabilities

TABLE IIB[a]

IMPLIED TAKE PROBABILITIES FOR THE CENTIPEDE GAME

	Session	p_1	p_2	p_3	p_4	p_5	p_6
Four Move	1 (PCC)	.06 (100)	.28 (94)	.65 (68)	.83 (24)		
	2 (PCC)	.10 (81)	.42 (73)	.76 (42)	.90 (10)		
	3 (CIT)	.06 (100)	.46 (94)	.55 (51)	.61 (23)		
	Total 1–3	.07 (281)	.38 (261)	.65 (161)	.75 (57)		
High Payoff	4 (CIT)	.15 (100)	.44 (85)	.67 (48)	.69 (16)		
Six Move	5 (CIT)	.02 (100)	.09 (98)	.44 (89)	.56 (50)	.91 (22)	.50 (2)
	6 (PCC)	.00 (81)	.02 (81)	.04 (79)	.49 (76)	.72 (39)	.82 (11)
	7 (PCC)	.00 (100)	.07 (100)	.15 (93)	.54 (79)	.64 (36)	.92 (13)
	Total 5–7	.01 (281)	.06 (279)	.21 (261)	.53 (205)	.73 (97)	.85 (26)

Games with incomplete information

In theory, applying backward induction presupposes that the players **take turns**.

But there are situations in which

- players move **simultaneously** at some point;
- some players have **information** that the others do not.

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Incentives and asymmetric information

When some players know things that others do not.

The market for used cars (Akerlof, 1970)

- Half the cars on the market are of high quality q_h, the other half of low quality q_l.
- Buyers are willing to buy only if the price does not exceed $\frac{3}{2}q$, but they **do not know** q.
- Sellers want to sell at a price of at least q_h if the quality is high, q_l otherwise.

Buyers observe quality

Sellers can sell at q.

Buyers do not observe quality

Buyers pay at most the average $\frac{1}{2} \cdot \frac{3}{2}q_h + \frac{1}{2} \cdot \frac{3}{2}q_l$.
High-quality sellers can sell only if

$$\frac{3}{4}q_h + \frac{3}{4}q_l \geq q_h \iff q_h \leq 3\,q_l.$$

If $q_h > 3\,q_l$, high-quality sellers cannot sell: **only sellers of low-quality cars remain** on the market.

Income taxation (Mirrlees, 1971)

- Citizens have a utility u and earn an income $r(e_i, \theta_i)$ that depends on their **effort** e_i and their **productivity** θ_i.
- The state chooses an income tax T so as to maximise $\sum_i u(r(e_i, \theta_i) - T(r(e_i, \theta_i)))$.

The state observes productivities

It can make the tax depend on productivity, and redistribute wealth without any further problem (cf. the theorems of welfare economics).

The state does not observe productivities

It must make taxes depend on income, so it chooses T to solve

$$\max_T \sum_i u(r(e_i, \theta_i) - T(r(e_i, \theta_i)))$$

s.t. $e_i \in \arg\max_e u\big(r(e, \theta_i) - T(r(e, \theta_i))\big)$.

Take-aways

- **Game theory** studies choices whose consequences depend on the choices of others. Players are rational and believe the others are rational too.
- A **Nash equilibrium** is a profile in which no one wants to deviate. It need not be a **Pareto optimum**: prisoner's dilemma, public goods, tragedy of the commons.
- Some games have no equilibrium in pure strategies, but Nash showed that an equilibrium in **mixed strategies** always exists.
- In the **laboratory**, many people cooperate, give and punish: altruism, reciprocity and inequality aversion challenge the self-interest hypothesis.
- **Coordination** relies on conventions and salient **focal points**, shaped by culture.
- In dynamic games, **backward induction** removes non-credible threats; with **asymmetric information**, markets and taxes run into incentive problems.